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ENGINEERING ELECTRICITY

HUDSON

WORKS OF

PROFESSOR RALPH G. HUDSON

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ENGINEERING ELECTRICITY

BY

RALPH G. HUDSON, S. B.

Assistant Professor of Electrical Engineering at the Massachusetts Institute of Technology

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PREFACE

This book was written primarily for technical students not specializing in electrical engineering but may also serve as an introductory text for electrical engineering students. It contains an outline of the lectures given by the author to junior and senior students at the Massachusetts Institute of Technology, specializing in civil, mechanical, mining, chemical and general engineering, physics, naval architecture and engineering administration respectively. The subject is discussed in twenty lectures extending in some courses over ten weeks and in others over twenty weeks. Each lecture is followed by one or more recitations and the assignment of relevant problems. An independent laboratory course covering the same ground follows the lecture course and in some cases is given simultaneously. In some instances the course is succeeded by other courses in which various industrial applications of electricity are discussed in greater detail.

Although the keynote of such instruction is brevity, it is believed, nevertheless, that it should be rigorous in view of the maturity of the students involved. An important aim of the course is to develop the reasoning power of the student rather than to encourage the accumulation of disconnected facts. The student while engaged in a course of mental training is expected to become familiar with the general principles of electrical engineering and their practical applications. Descriptive matter is included only to the extent that the student may visualize the practical devices which illustrate applications of the fundamental principles. Recourse is made to mathematical methods of development only when the direct exposition of the subject appears to be more involved.

The course is intended to cover the general principles of electricity and magnetism most frequently applied in engineering practice. The principles of electrochemistry, illumination, electrostatics and the wireless propagation of energy are not considered for obvious reasons in such a short course. No use has been made of footnotes and appendices and the general reading matter has been reduced to a minimum so that the student may give his undiverted attention to the understanding of the basic principles. Most of the formulas are derived so that they may be more clearly understood. The recitation periods are devoted principally to questions and problems designed to interpret and expand the text. While the text may be used by others with a different educational objective the author's use of the text is described briefly so that the reader may understand the educational motive which prompted its preparation and arrangement.

The photographic reproductions of electrical apparatus appearing in Chapter XVII were supplied generously by the General Electric Company, the Westinghouse Electric and Manufacturing Company, the Crocker-Wheeler Company, the Weston Electrical Instrument Company, the Leeds and Northrup Company and the General Radio Company.

RALPH G. HUDSON.

CAMBRIDGE, MASS.
June, 1920.

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Engineering Electricity

CHAPTER I

DIRECT CURRENT CIRCUITS

Engineering Requisites. All engineering operations involve the use of a material substance (matter), an active agent (energy), and a directive influence (the human mind). Each of these requisites is identified by certain properties or manifestations. Matter possesses weight and inertia, energy can perform work, and the human mind can instigate and prescribe to a certain degree the kind of work that energy is to perform. In many cases physical phenomena also suggest the existence of a pervasive medium (ether).

Present State of Man's Intelligence. While future discoveries may establish a closer identity between matter, energy, and ether, the engineer of the present may only comprehend certain elements and compounds of matter, certain forms of energy, and a non-material medium, all controlled to some extent by his mind. With the aid of energy man's mind enables him to construct compounds from elements, derive elements from compounds, change the shape of material bodies, change their state of rest or motion, and convert one form of energy into another form either completely or in part.

Forms of Energy. Energy was first recognized in the obvious forms associated with bodies in motion or under the influence of gravity. The same agent was subsequently found to exist in the more latent forms of thermal and chemical energy. Electric and magnetic forms of energy were discovered last because our senses are less responsive to these forms than to the others. Electric and magnetic energy differ from other forms of energy in that they may exist apart from matter and may be propagated through space free from matter.

Electric Current. The economic value of electric energy is due principally to the possibility of transmitting this form of energy in large quantities over great distances without considerable loss. The propagation of electric energy through space along a wire is accompanied by certain manifestations located within the wire or in the surrounding space. These manifestations associated with the propagation of electric energy are ascribed to a flow of electricity through the wire and this flow of electricity is called an electric current.

The Effects of an Electric Current. The principal effects of an electric current are the thermal, chemical, magnetic, and force effects. The thermal effect is manifested by the heating of certain materials and the heating or cooling of the junction between certain materials conducting an electric current; the chemical effect by the chemical reaction associated with the flow of current through certain materials, chiefly liquids; the magnetic effect by the associated magnetic energy stored in the space surrounding a current; the force effect by the force exerted upon a current when placed in space containing magnetic energy.

Definition of the Ampere. The magnitude of an electric current may be defined arbitrarily in terms of any one of its effects. While the force effect was first chosen for such definition the chemical effect proved more convenient and the unit of current, the ampere, is now defined as that strength of current which will deposit 1.11800 milligrams of silver per second from an aqueous solution of pure silver nitrate (see Fig. 78 on page 66). The direction of the current is assumed to be toward the surface on which the silver is deposited. The symbol for current is I and its direction is indicated by the horizontal or vertical projection of an arrow as shown in Fig. 1.

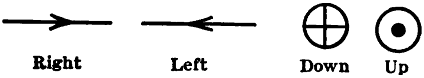


Fig. 1

Energy Conversion Associated with an Electric Current. The flow of an electric current through any part of an electric circuit is accompanied by a continuous conversion of energy from one form into another. The nature of the conversion in any part depends upon its material structure, the constancy of the current, the lateral motion of the current in magnetic space, and the fixity of the magnetic energy in the surrounding space. The flow of an electric current in most substances is accompanied by a conversion of electric into thermal energy and, in the case of certain dissimilar metals in contact, by a conversion of electric to thermal energy or the reverse. In certain materials, especially liquids, an additional conversion from electric to chemical energy or the reverse takes place. A variable current is accompanied by a conversion from electric to magnetic energy or the reverse, and lateral motion of a current in magnetic space may be accompanied by a conversion of electric into mechanical energy or the reverse.

Definition of Electromotive Force. Each of the cases in which the energy conversion is stated to be reversible may be utilized either as a source of electric energy or as a receiver of electric energy. Any device possessing the property of reversible conversion is called a reversible converter. The rate of reversible conversion in most reversible converters is directly proportional to the current. The rate of reversible conversion, or the reversible power, per unit current is called the electromotive force of the converter. When the power is measured in watts and the current in amperes the electromotive force is measured in volts. Electromotive force is commonly abbreviated "e.m.f." and is given the symbol E. Hence the e.m.f. of any device in which a current of I amperes is accompanied by a reversible conversion at a rate of P watts is given by

1 $$E = \frac{P}{I} \text{ volts.}$$

The direction of the e.m.f. in any reversible converter is assumed to be in the direction of the current if the converter is a source of electric power and in the opposite direction to the current if the converter is a receiver of electric power. A reversible converter of constant e.m.f. is represented by the symbol $\overset{\rule{1em}{2pt}}{\rule{3em}{0.5pt}}$ and it is assumed that the e.m.f. acts within such a converter from the short line to the long line.

Definition of Resistance. The order of the conversion of energy in many substances, especially solids, is independent of the direction of the current (electric energy being converted into thermal energy with either direction of the current) and the rate of conversion in such substances is proportional to the square of the current. In an irreversible converter the rate of conversion per unit of current is therefore a variable but the ratio of the rate of conversion to the current squared is a constant and is called the resistance of the irreversible converter. When the power is measured in watts and the current in amperes the resistance is measured in ohms. The symbol for resistance is R and an irreversible converter is represented by the symbol -/\/\/\-. Hence the resistance of any device in which electric energy is converted into thermal energy with either direction of the current at a rate of P watts when the current is I amperes is given by

2 $$R = \frac{P}{I^2} \text{ ohms.}$$

The resistance of any device may be expressed in microhms (one microhm equals one-millionth of an ohm), or in megohms (one megohm equals one million ohms).

Rate of Energy Conversion or Power. The reversible power converted in any device of e.m.f. E volts when conducting a current of I amperes, from 1, is given by

3 $$P = EI \text{ watts.}$$

Positive power indicates conversion into electric power and negative power conversion from electric power.

The irreversible power converted in any device of resistance R ohms when conducting a current of I amperes, from 2, is given by

4 $$P = I^2R \text{ watts.}$$

This power is negative since electric power is always converted into thermal power in an irreversible converter.

Definition of Potential or Voltage. The electric power delivered from any part of an electric circuit confined between two points a and b (Fig. 2), from 3 and 4, is given by

5 $$P_{ab} = +EI - I^2R \text{ watts.}$$

The power delivered per ampere from any part of an electric circuit confined between two points a and b is called the potential or voltage between the two points. Dividing 5 by I, the potential or voltage (V) between the two points a and b in Fig. 2 is then given by

6 $$V_{ab} = +E - IR \text{ volts},$$

where E is positive if directed toward b and I is positive if flowing toward b. A positive value of V_{ab} indicates a potential rise toward b and a negative value a potential drop toward b.

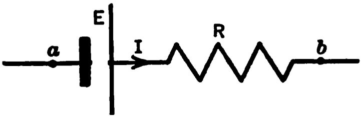


FIG. 2

The power delivered from any part of a circuit, by definition of V, is given by

7 $$P = VI \text{ watts}.$$

In 7, V is positive for a potential rise in the direction of the current and is negative for a potential drop in the direction of the current. Positive power indicates electric power delivered from, and negative power, electric power delivered to, the part of the circuit under consideration.

Comparative Meaning of E.m.f. and Potential. The e.m.f. in any part of an electric circuit indicates the rate of energy conversion per ampere in that part of the circuit either from electric energy to some other form or the reverse, depending upon the relative direction of the current and the e.m.f. The magnitude or the direction of the e.m.f. does not depend upon the magnitude or the direction of the steady current unless the current changes the physical state of the converter. The potential or voltage between two points in any circuit indicates the rate of energy per ampere delivered or received between the two points. The magnitude or the direction of the potential between two points may and usually does depend upon the magnitude and direction of the current flowing between the two points. No current will flow in any circuit which contains no e.m.f. but current may flow in a circuit in which the potential between any two points is zero. The electromotive force of any device indicates the power converted per ampere within that device exclusive of the power converted into heat in the device due to the passage of current through its material substance. The potential between the terminals of any devic indicates the power per ampere delivered to or received from th device in any form.

Energy Converted in Any Device. The energy converted t any form in any device is given by

$$W = \frac{VIh}{1000} \text{ kilowatt-hours,} \qquad (8)$$

where V is the potential between the terminals of the device, I is the current flowing through the device, and h is the number o hours that the device is electrically connected.

The energy that is converted from electric energy to any othe form or the reverse depending upon the direction of the current in any device is given by

$$W = \frac{EIh}{1000} \text{ kilowatt-hours,} \qquad (9)$$

where E is the e.m.f. of the device. (I and h are described under 8.)

The energy converted into heat in any device with either direction of the current is given by

$$W = \frac{I^2Rh}{1000} \text{ kilowatt-hours,} \qquad (10)$$

where R is the resistance of the device. (I and h are described under 8.)

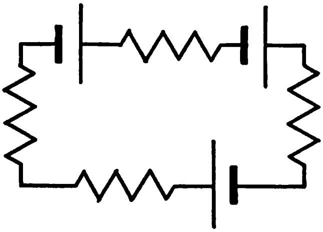


FIG. 3

Series and Parallel Connections. When the various parts of an electric circuit are connected end to end successively, as shown in Fig. 3, it is called a series connection. The current throughout a series connection is the same in each part. If some of the parts of a circuit are connected end to end jointly, as shown in Fig. 4, it is called a parallel connection. The potential between the terminals of each part of a parallel connection is the same for each part.

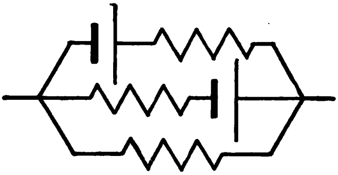


FIG. 4

Properties of the Series Connection. The resultant e.m.f. in a stated direction in a series connection equals the algebraic sum of its constituent e.m.f.'s., an e.m.f. acting in the stated direction being positive and in the opposite direction negative. Hence

11 $$E = \pm E_1 \pm E_2 \pm E_3 \pm \text{etc. volts.}$$

The resultant resistance of a series connection equals the arithmetic sum of its constituent resistances. Hence

12 $$R = R_1 + R_2 + R_3 + \text{etc. ohms.}$$

The potential rise in a stated direction between the ends of a series connection equals the algebraic sum of the potentials between the ends of its constituent parts, a potential rise in the stated direction being positive and a potential drop negative.

Hence, in Fig. 5,

13 $$V_{ad} = \pm V_{ab} \pm V_{bc} \pm V_{cd} \text{ volts.}$$

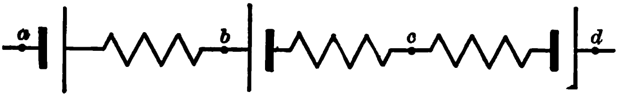


FIG. 5

The current flowing in any series connection (Fig. 2), from 6, is given by

14 $$I = \frac{+E - V_{ab}}{R} \text{ amperes.}$$

All quantities represented in 14 apply only to a single series connection located between a and b. E is the resultant e.m.f. of the connection between a and b, V_{ab} is the potential between a and b, R is the resultant resistance of the connection between a and b, and I is the current flowing from a to b. The convention of signs is the same as that described under 6. In a continuous series connection a and b are identical points and V_{ab} equals zero.

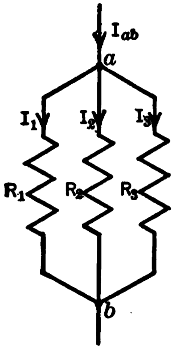


FIG. 6

Properties of the Parallel Connection. In any parallel connection the sum of the currents flowing toward any junction equals the sum of the currents flowing away from that junction. In Fig. 6, it may then be stated that

15 $$I_{ab} = I_1 + I_2 + I_3 \text{ amperes.}$$

Parallel Connections containing No E.m.f.'s. If no part of a parallel connection contains an e.m.f. it follows, from 14. that in Fig. 6

$$I_{ab} = \frac{V_{ab}}{R_{ab}}, \quad I_1 = \frac{V_{ab}}{R_1}, \quad I_2 = \frac{V_{ab}}{R_2}, \quad \text{and} \quad I_3 = \frac{V_{ab}}{R_3}.$$

Then, from 15,

$$\frac{V_{ab}}{R_{ab}} = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3} \quad \text{and}$$

16 $$\frac{1}{R_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R_{ab} is the equivalent resistance of a parallel connection containing no e.m.f. When the parallel connection contains only two parts, $R_{ab} = \frac{R_1 R_2}{R_1 + R_2}$ and when n parts, each of resistance R, are connected in parallel, $R_{ab} = \frac{R}{n}$. From 6 it also follows that in Fig. 6

17 $$I_1R_1 = I_2R_2 = I_3R_3 = I_{ab}R_{ab}.$$

In any circuit containing parallel connections without e.m.f.'s the circuit as a whole may be converted into a simple series circuit for the purpose of calculation by substituting for each of its parallel connections the equivalent resistance given by 16.

Parallel Connections containing E.m.f.'s. When two or more parts of any circuit are not connected in series and contain e.m.f.'s, the resultant e.m.f. or resultant resistance of the circuit cannot be calculated and the determination of the magnitude and direction of the current in each part involves the solution of two or more simultaneous equations based upon 6 and 15.

Example. The currents flowing in the various parts of the circuit shown in Fig. 7 are indicated arbitrarily in magnitude and direction by symbols and arrows. Then between the two points a and b, from 6,

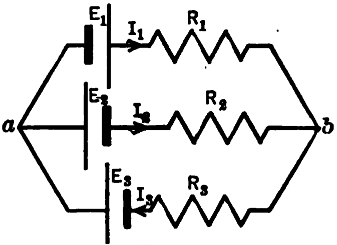


FIG. 7

$$(1)\quad +E_1 - I_1R_1 = -E_2 - I_2R_2$$
$$(2)\quad -E_2 - I_2R_2 = -E_3 + I_3R_3$$

and from 15,

$$(3)\quad I_1 + I_2 = I_3.$$

The magnitude and actual direction of each of the currents may be determined by solving the three simultaneous equations, a positive value of current in any case indicating that the direction assumed in the figure is correct and a negative value indicating a direction opposite to that in the figure.

Relative Resistances of Materials. Among the elements, silver has the lowest resistance per unit volume. Other metals, such as copper, aluminum, iron, etc., also possess relatively low resistance and are called conductors. Various materials, such as glass, porcelain, slate, rubber, etc., possess relatively high resistance and are called insulators. All materials conduct current, however, and the distinction between conductors and insulators is made only to express their relative resistances per unit volume.

The American Wire Gage. The length of electric wires is usually measured in feet, the diameter in mils (1 mil equals 0.001 inch) and the area in circular mils (area in circular mils equals the square of the diameter in mils). Wires are drawn for general use in various sizes represented by the American Wire Gage in which the consecutive sizes differ in area by approximately 26 per cent. A copper wire table giving the constants of wires suitable for light and power circuits will be found on page 184.

Resistance Calculations. The resistance of any wire of uniform cross-section is proportional to its length and inversely proportional to its cross-sectional area. Given the resistance (R_1), length (l_1), and area (A_1) or weight (W_1) of any wire, the resistance of any other wire of the same material of length (l_2), area (A_2) or weight (W_2) is determined by

18 $$\frac{R_1}{R_2} = \frac{l_1A_2}{l_2A_1} = \frac{l_1^2W_2}{l_2^2W_1}.$$

Any units of length, area or weight may be employed, provided the same units are used for both wires. If the given wire is one foot long and one mil in diameter it is called a mil-foot of wire and its resistance is called the resistivity (ρ) per mil-foot of the material. Given the resistivity (ρ) of any material per mil-foot the resistance of any wire of the same material l feet in length and A circular mils in cross-sectional area is given by

19 $$R = \frac{\rho l}{A} \text{ ohms.}$$

The resistivity of a material may also be expressed in microhms per cm. cube, megohms per cm. cube or ohms per meter-gram.

The resistivity of various materials in microhms per cm. cube and megohms per cm. cube is given on page 185.

Variation of Resistance with Temperature. The resistance of all materials changes with the temperature and is assumed to be zero for all materials at zero degrees absolute. The resistance (R_2) of any material at t_2° C. which has a resistance of R_1 ohms at t_1° C. is given by

20 $$R_2 = R_1\,[1 + \alpha_1(t_2 - t_1)] \text{ ohms,}$$

where α_1 is the resistance-temperature coefficient of the material at t_1° C. Values of α for various materials are given on page 185. The resistance-temperature coefficient of annealed 100 per cent conductivity copper at any temperature is given by

21 $$\alpha_t = \frac{1}{234.5 + t}.$$

The temperature (t_2) of a material of R_2 ohms resistance which has a resistance of R_1 ohms and a resistance-temperature coefficient of α_1 at t_1° C., from 20, is given by

22 $$t_2 = \frac{R_2 - R_1}{\alpha_1 R_1} + t_1 \text{ degs. C.}$$

For annealed 100 per cent conductivity copper

23 $$t_2 = \left(\frac{R_2 - R_1}{R_1}\right)(234.5 + t_1) + t_1 \text{ degs. C.}$$

Insulation of Wires. Wires are insulated with rubber for general use, except (1) when subjected to temperatures in excess of 50 degs. C., (2) when exposed to the action of corrosive vapors and (3) when conditions prescribe the use of a fire-proof insulation. Impregnated cotton braid is employed to insulate wires in low-voltage machinery, in buildings when wires are not concealed and in the open when wires are closely spaced. Wires subjected to high temperatures are sometimes insulated with asbestos. Wires drawn in underground conduits are commonly insulated with varnished cambric or impregnated paper protected by a lead sheath. Mica insulation is used to a considerable extent in high-potential machinery. Transmission line wires are usually bare and are supported by glass, porcelain or composition insulators. The carrying capacity of insulated wires or the maximum current that insulated wires may carry continuously without overheating and injuring the insulation depends upon the nature of the insulating material. The values recommended for insulated copper wires by the National Board of Fire Underwriters are given on page 184.

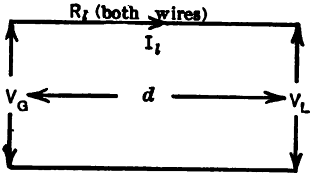


FIG. 8

Transmission Line Calculations. The principles stated on the previous pages may be applied to the transmission line (Fig. 8) as follows: V_G and V_L are the respective potentials at the generator and load ends of the line, R_l is the resistance of both line wires, d is the distance in feet from the generator to the load, I_l is the line current, and P_G and P_L are the power delivered to the line and received from the line respectively. From 13,

24 $$V_L = V_G - I_l R_l \text{ volts.}$$

From 7,

25 $$P_G = V_G I_l \text{ watts.}$$

26 $$P_L = V_L I_l \text{ watts.}$$

From 7 and 4,

27 $$P_L = V_G I_l - I_l^2 R_l \text{ watts.}$$

The efficiency of transmission or the ratio of the power transmitted to the load to the power delivered to the line is given by

28 $$\eta = \frac{V_L I_l}{V_G I_l} = \frac{V_L}{V_G}.$$

From 27,

29 $$I_l = \frac{V_G \pm \sqrt{V_G^2 - 4\,R_l P_L}}{2\,R_l} \text{ amperes.}$$

From 24 and 19,

30 $$A = \frac{2\,\rho I_l d}{V_G - V_L} \text{ circular mils.}$$

From 26 and 30,

31 $$A = \frac{2\,\rho\,dP_L}{(V_G - V_L)\,V_L} \text{ circular mils.}$$

The weight of a copper wire of length, l feet, and area, A circular mils, is given by

32 $$G = 3.03 \times 10^{-6}\,lA \text{ pounds.}$$

From 32 and 31, the weight of a copper transmission line is given by

33 $$G = \frac{1.28 \times 10^{-4} P_L d^2}{(V_G - V_L)\,V_L} \text{ pounds.}$$

Economics of the Transmission Line. The annual cost of the energy lost in a transmission line of R_l ohms resistance which conducts a steady current of I_l amperes for h hours per year, from 10, is $\frac{I_l^2 R_l hc}{1000}$ dollars, where c is the cost of the electric energy in dollars per kilowatt-hour. The annual investment charge associated with the line wires is $\frac{pGc'}{100}$ dollars, where p is the annual percentage rate of interest on the capital invested in the line wires which will pay the annual capital interest, taxes and depreciation of the line wires, G is the weight of the copper wire in pounds and c' is the cost of the wire in dollars per pound. The principle known as Kelvin's Law indicates that the annual operating expense of a transmission line is a minimum when the annual cost of the lost energy is made equal to the annual investment charge associated with the wire. By equating the above costs, substituting 19 for R_l and 32 for G, the most economical section of copper wire for a transmission line is given by

34 $$A = 593\, I_l \sqrt{\frac{ch}{c'p}} \text{ circular mils.}$$

Potential Limitations in the Two-Wire System. The power lost in a transmission line, from 4, is $I_l^2R_l$ watts. Since I_l, from 25, equals $\frac{P_G}{V_G}$ amperes, the power lost in a transmission line is also given by $\frac{P_G^2R_l}{V_G^2}$ watts. The power lost in the line is inversely proportional to the square of the generator potential and the generator potential should therefore be made as high as possible. The magnitude of the potential impressed between the terminals of devices receiving energy from a direct-current circuit is limited for various reasons in the case of motors to approximately 600 volts and in the case of incandescent lamps to approximately 115 volts. The operation of these devices in series is in general undesirable. Motors connected in series are difficult to insulate and their operation in series under most circumstances is unstable. The respective luminosities of incandescent lamps connected in series would vary unless the lamps were selected with care, an increase of resistance in any lamp would shorten its length of service, one lamp could not be lighted alone, the failure of one lamp would extinguish the others unless an automatic shunt was attached to each lamp, and the higher potential associated with such a connection would increase the danger of shock to those who might come in contact with the circuit.

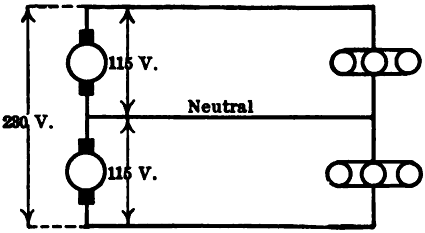


Fig. 9

The Edison Three-Wire System. If the lamps in any incandescent lighting system are divided into two groups and these groups are connected to a three-wire system as shown in Fig. 9, all of the disadvantages of the series connection noted above are eliminated. The current in the neutral wire will be zero if the current in each group of lamps is made the same and the efficiency of transmission will equal that of a two-wire 230-volt system. Danger of shock is removed by grounding the neutral wire so that the potential to ground from either outside wire is limited to 115 volts. A balanced three-wire system designed to replace a two-wire system (the power transmitted, length of line, power loss in the line and potential at the lamps being the same in each case) requires a weight of copper equal to 37.5 per cent of that required by the two-wire system. Although the neutral wire of a balanced system conducts no current it is usually made equal in size to either outside wire so that it may in an emergency carry as much current as either outside wire. Motors supplied from a three-wire system are usually connected between the outside wires.

CHAPTER II

ELECTROMAGNETISM

The Magnetic Field. A change in the magnitude of the current flowing in any circuit is accompanied by a conversion of electric into magnetic energy or the reverse; with an increasing current electric energy is converted into magnetic energy and with a decreasing current magnetic energy is converted into electric energy. The magnetic energy thus converted from electric energy with an increasing current is distributed throughout the space or the field surrounding the circuit and manifests itself by a force action on an electric current or on certain materials located in that space. Any region in which such force action is manifested on an electric current or on certain materials is called a magnetic field.

Magnetic Flux. The force action associated with a magnetic field is ascribed to the reaction between the electric current placed in the field and a magnetic flux which is assumed to permeate the field of magnetic energy. The force action on certain materials placed in a magnetic field is attributed to the same reaction, the current reacted upon in this case being concealed within the atomic structure of the material. Unit magnetic flux may be visualized as a continuous tube of variable cross-section and the path of such a tube may be indicated by a line which represents the axis of the tube. A single tube of magnetic flux or unit magnetic flux is called a maxwell and the axis of such a tube is called a line of flux. It should be noted that the flux permeating an area smaller than the cross-section of a single tube at any point may be less than one maxwell and that the flux permeating any area will also depend upon the position of the area in space.

Magnetic Flux Density. The force acting upon a current placed in a magnetic field is proportional to the magnitude of the current, the length of the wire conducting the current, the position of the wire in the field, and the strength of the magnetic field. The conception of magnetic flux suggests that its density be taken as a measure of the strength of the magnetic field. Since the force action on an electric current varies directly with the strength of the magnetic field this force action may be taken as an arbitrary measure of the magnetic flux density at any point in the field. If a wire s centimeters in length conducting a current of I amperes is placed in a magnetic field of uniform strength along the wire and the force acting normal to the wire is F dynes the magnetic flux density normal to the plane of the wire and the force is defined by

35 $$B = \frac{10\,F}{Is} \text{ gausses.}$$

Unit magnetic flux density, the gauss, equals one maxwell per square centimeter. The relative directions of the flux density

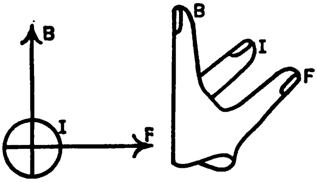


Fig. 10

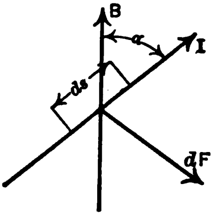


Fig. 11

(B), current (I) and force (F) are shown in Fig. 10. If the directions of the current and the force are indicated respectively by the middle finger and the thumb of the left hand the direction of the flux density normal to the plane of the current and the force is indicated by the forefinger. The direction of the magnetic flux at any point in space is normal to that plane of the current and the force in which the force acting on the current is a maximum. The flux density at a point is a maximum in the direction of the magnetic flux at the point.

Force on a Current placed in a Magnetic Field. The force acting normal to a current element ds centimeters in length and conducting a current of I amperes placed in a magnetic field of B gausses maximum flux density when the directions of the current and the flux density differ by the angle α (Fig. 11) is given by

36 $$dF = \frac{BI\,ds \sin \alpha}{10} \text{ dynes.}$$

Flux Density due to a Current Element. The flux density at a point in space due to a current may be determined experimentally by measuring the force exerted upon another current placed at the point in question, the flux density for any observed force being calculated by 35. An exploration of the magnetic field surrounding an electric current indicates that the flux density at any point (Fig. 12) is proportional to (1) the current (I) which has established the magnetic flux, (2) the length of the current path (ds), (3) the sine of the angle (α) between the extension of the line joining the point and the current element and the direction of the current, (4) the character of the medium in which the point is located (this magnetic property of the medium is called its permeability (μ)), and is inversely proportional (5) to the square of the distance (r) from the point to the current element. If the current is measured in amperes and the length of the current path and the distance from the point to the current element in centimeters the flux density at the point is given by

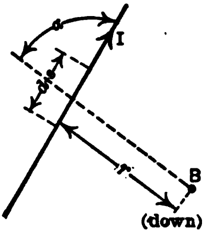


FIG. 12

37 $$dB = \frac{I\mu \sin \alpha \, ds}{10 r^2} \text{ gausses.}$$

The direction of the flux density is perpendicular to the plane of the current element and the point (Fig. 13), and is clockwise in direction when the current flows away from the observer and counterclockwise in direction when the current flows toward the observer.

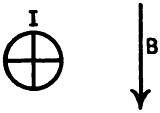


FIG. 13

Permeability of a Material. The flux density due to a current at a point located in space filled consecutively with various materials is sensibly the same for most materials but is materially increased when certain materials fill the space; notably, iron, cobalt, nickel and Heusler's alloy (manganese, aluminum and copper or zinc). The increased flux density resulting from the introduction of any material into the space in which the point is located is attributed to a magnetic property of the material, called its permeability. The permeability of any substance is measured by the ratio of the flux density at a point in space filled by the material in question to the flux density at the same point in air due to the same current. Hence, the permeability of any substance may be defined by the ratio,

38 $$\mu = \frac{B_{\text{material}}}{B_{\text{air}}}.$$

The flux density in air is frequently given the symbol (H). The permeability of most materials is very nearly one and such materials are called non-magnetic. All materials of permeability greater than one are called magnetic and the permeability of such materials may be as high as 25,000 for electrolytic iron, 7000 for commercial soft iron, 200 for nickel and Heusler's alloy, and 175 for cobalt. The permeability of all magnetic materials is a variable and depends upon the initial flux density in the same space filled with air prior to the introduction of the magnetic material. The permeability of all magnetic materials is comparatively low when placed in a magnetic field in air of high flux density. When certain materials are placed in space formerly occupied by air the flux density in this space is decreased. Materials possessing this property (bismuth is the best example) are called diamagnetic and the permeability of such materials is less than one.

Flux Density due to a Current Path of Any Shape. The flux density produced at a point in space by a current path of any length or shape may be determined in any case by integrating the flux densities at the point due to the elements of the current path. The results of such integrations of 37 are given for certain cases as follows:

Case I. Flux density at a normal distance of r centimeters from the axis and outside of a straight wire (Fig. 14) conducting I amperes and surrounded by a medium of permeability (μ).

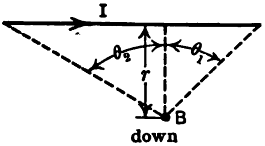


FIG. 14

39 $$B = \frac{I\mu}{10r}(\sin\theta_1 + \sin\theta_2) \text{ gausses.}$$

Case II. Flux density on the axis of a single turn of wire (Fig. 15) r centimeters in radius conducting I amperes and surrounded by a medium of permeability (μ).

40 $$B = 0.628\, Ir\mu \sin^3\theta \text{ gausses.}$$

Case III. Flux density on the axis near the middle of a long solenoid (Fig. 16) wound uniformly with n turns of wire per centimeter length of axis, each turn conducting I amperes and surrounded by a medium of permeability (μ).

41 $$B = 1.26\, nI\mu \text{ gausses.}$$

Case IV. Flux density at a point distant r centimeters from the axis of and within a toroidal coil (Fig. 17) of N turns conducting a current of I amperes and wound upon a toroidal core of permeability (μ).

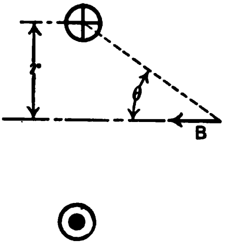


FIG. 15

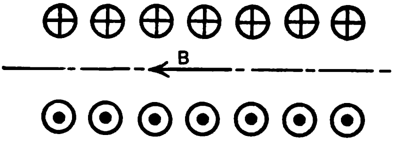


FIG. 16

$$B = \frac{0.2\,NI\mu}{r}\ \text{gausses.} \tag{42}$$

Energy converted by a Variation in the Flux linking a Current. The force acting on a wire (Fig. 18) ds centimeters in length conducting a current of i amperes and placed in a plane normal to a magnetic flux of B gausses flux density, from 36, is given by $dF = \frac{Bi\,ds}{10}$ dynes. The direction of the force, from Fig. 10, is toward the right. The work done in moving the wire in the plane to the left a distance of dx centimeters is given by $dw = \frac{Bi\,ds\,dx}{10}$ ergs.

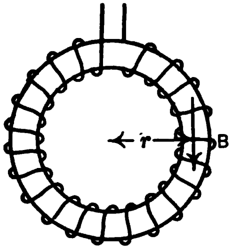


FIG. 17

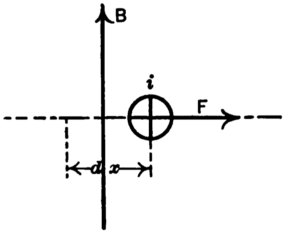


FIG. 18

Since a magnetic field is permeated by continuous tubes of flux the change of flux ($d\phi$) surrounding or linking the displaced wire is $B\,ds\,dx$ maxwells. The work done on the wire during its displacement is then given by $dw = \frac{i\,d\phi}{10}$ ergs and represents the energy converted when the flux linking a current is changed by $d\phi$ maxwells. The energy converted by changing the flux linking N turns of wire is therefore given by

43 $$dw = \frac{Ni\,d\phi}{10} \text{ ergs.}$$

During an increase of flux the electric circuit loses energy and during a decrease of flux the electric circuit gains energy. If the current (i) in 43 is maintained at a constant strength of I amperes while the flux linking the N turns is changed by a finite amount $\Delta\Phi$ maxwells, the energy converted by the change in flux, from 43, is given by

44 $$w = \frac{NI\,\Delta\Phi}{10} \text{ ergs.}$$

If the current (i) in 43 varies in strength while the flux linking the N turns is changed, the energy converted during a simultaneous change of current and flux is given by $w = \frac{N}{10}\int i\,d\phi$. When the flux linking an electric circuit is established by the current flowing in the circuit, the flux may be expressed by some function of the current and the energy converted during any change in the current may be determined by integration.

The flux linking any circuit located in a medium of constant permeability, from 37, is directly proportional to the current, or $\phi = Ki$. Since $d\phi = K\,di$, $w = \frac{NK}{10}\int i\,di$ and $w = \frac{NKi^2}{20}$, the limits being the initial and the final values of the current. The energy converted when the current in N turns of wire is increased from zero to I amperes is then given by $w = \frac{NKI^2}{20}$ ergs and, since $\Phi = KI$, the energy converted is also given by

45 $$w = \frac{N\Phi I}{20} \text{ ergs.}$$

The establishment of a current in any circuit is thus accompanied by a conversion of electric into magnetic energy, the amount of energy converted in any case being given by an integration of 43, and when the permeability of the surrounding medium is constant, by 45. When the current in any circuit is interrupted the magnetic energy stored in the surrounding medium is again converted into electric energy, either in whole or in part, depending upon the nature of the surrounding medium.

Space Distribution of Magnetic Energy. The magnetic energy (W) stored within the internal core of a toroidal coil (Fig. 17) of N turns conducting a current of I amperes (the permeability of the core being assumed constant), from 45, is $\frac{N\Phi I}{20}$ ergs. The volume (V) of a toroidal core of r centimeters mean radius and A square centimeters cross-sectional area when the area is small compared with the radius of the core is $2\pi rA$ cubic centimeters. The magnetic energy per cubic centimeter (w) stored in the toroidal core is therefore given by $w = \frac{N\Phi I}{20 \cdot 2\pi rA} = \frac{NBI}{40\pi r}$ ergs per cubic centimeter. The flux density in such a core, from 42, is given by $B = \frac{0.2\,NI\mu}{r}$ gausses. Substituting for I its equivalent, $\frac{5\,Br}{N\mu}$, $w = \frac{NB}{40\pi r} \cdot \frac{5\,Br}{N\mu}$ and the magnetic energy per cubic centimeter stored in any medium of constant permeability is given by

46 $$w = \frac{B^2}{8\pi\mu} \text{ ergs per cubic centimeter.}$$

Magnetomotive Force and Reluctance. Since the unit of flux, the maxwell, may be represented in space by a continuous tube of variable cross-section the magnetic flux or the aggregate of the constituent tubes of flux may also be represented in space by a continuous volume of variable cross-section. The magnetic energy (W_1) stored in any longitudinal section of this volume, l_1 centimeters in length, A_1 square centimeters in cross-sectional area, and filled with a medium of constant permeability (μ_1) (multiplying w in 46 by the volume of the section) is given by $W_1 = \frac{B_1^2 l_1 A_1}{8\pi\mu_1}$ ergs. The total magnetic energy stored in the magnetic field surrounding a circuit of N turns conducting a current of I amperes (the permeability of the surrounding medium being constant), from 45, is given by $W = \frac{N\Phi I}{20}$ ergs and must equal the sum of the energys stored in the constituent parts of the magnetic field. Hence $\frac{N\Phi I}{20} = \frac{B_1^2 l_1 A_1}{8\pi\mu_1} + \frac{B_2^2 l_2 A_2}{8\pi\mu_2} +$ etc. ergs. While the flux density throughout a magnetic field may vary from point to point the flux through any cross-section of the magnetic circuit is constant. Hence $\Phi = B_1A_1 = B_2A_2$, etc.

and
$$\frac{N\Phi I}{20} = \frac{\Phi^2 l_1}{8\pi\mu_1 A_1} + \frac{\Phi^2 l_2}{8\pi\mu_2 A_2} + \text{etc.}$$

Then
$$1.26\, NI = \Phi\left(\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \text{etc.}\right)$$

or

47
$$\Phi = \frac{1.26\, NI}{\dfrac{l_1}{\mu_1 A_1} + \dfrac{l_2}{\mu_2 A_2} + \text{etc.}} \text{ maxwells.}$$

In the continuous series electric circuit, the current equals the ratio of the electromotive force to the resistance. The analogous arrangement in 47 suggests that the numerator be called the magnetomotive force and the denominator the reluctance of the magnetic circuit. Magnetomotive force (abbreviated m.m.f.) is given the symbol ($\mathcal{F}$) and is measured in gilberts. Reluctance is given the symbol ($\mathcal{R}$) and is measured in oersteds.

The resultant magnetomotive force in a stated direction of several independent coils of wire wound upon the same magnetic circuit and conducting different currents is given by

48
$$\mathcal{F} = 1.26\,(\pm N_1I_1 \pm N_2I_2 \pm \text{etc.}) \text{ gilberts,}$$

where a positive or negative sign indicates that the associated magnetomotive force is acting respectively in the same or in the opposite direction to the stated direction.

The resultant reluctance of several longitudinal sections of a magnetic circuit of various respective lengths, cross-sectional areas and permeabilities is given by

49
$$\mathcal{R} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \text{etc. oersteds.}$$

The reluctance of several longitudinal sections of a magnetic circuit connected in parallel and containing no magnetomotive forces is given by 16, substituting $\mathcal{R}$ for R. The magnetic potential (G_{ab}) between two points in a magnetic circuit is given by 6, substituting G_{ab} for V_{ab}, $\mathcal{F}$ for E, Φ for I, and $\mathcal{R}$ for R. When certain parts of a magnetic circuit contain magnetomotive forces and are not connected in series the magnitude and direction of the flux in each part may be determined as indicated on page 9, substituting $\mathcal{F}$ for E, Φ for I, and $\mathcal{R}$ for R.

The Magnetization Curve. The magnetomotive force required to establish a stated flux in a magnetic circuit of constant cross-sectional area (A sq. cms.), length (l cms.) and uniform permeability (μ), from 47, is given by $1.26\ NI = \Phi \dfrac{l}{\mu A}$. The number of ampere turns required per centimeter length of the magnetic circuit is therefore given by

50 $nI = 0.793 \dfrac{B}{\mu}$ ampere turns per centimeter length of the magnetic circuit.

Since the permeability of all magnetic materials varies with the flux density equation 50, when applied to such materials, contains three variables, nI, B, and μ. If the corresponding values of B and nI are plotted for any material from test data nI may be determined easily without consideration of the value of μ. The curve obtained from such plotted values of B and nI for any material is called the magnetization curve (Fig. 19) of the material. Magnetization curves of various materials are given on page 186. It will be noted that when nI is small, B increases sensibly in proportion to nI, but for higher values of nI B increases less rapidly due to the decreasing permeability of the material. For all points above S on the curve the material is said to be saturated and for all points below S, unsaturated. The permeability of any material for any value of nI may be determined from its magnetization curve by substituting the corresponding value of B in 50 and solving for μ.

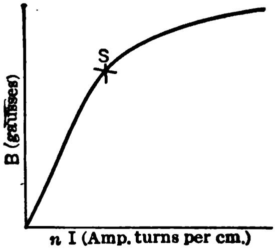


FIG. 19

The Hysteresis Cycle. If the magnetomotive force per centimeter applied to any magnetic material is increased until the flux density reaches any value (such as $+B_m$ in Fig. 20) and is then reduced to zero, the values of B corresponding to the decreasing values of nI will be represented typically by the curve connecting $+B_m$ with B_p. At the point B_p the material retains a part of its former flux density in the absence of any external magnetomotive force and is called a permanent magnet. The property of any material by which it retains a part of its former magnetic flux after the magnetomotive force is removed is called its retentivity. The greatest retentivity in magnetic materials is found in hard steels and the least in soft iron.

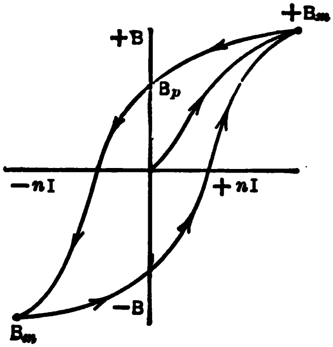


FIG. 20

The magnetic energy converted from electric energy during the application of an increasing magnetomotive force to a non-magnetic material (such materials possess no retentivity) is again converted into electric energy when the magnetomotive force is removed. In a magnetic material part of the magnetic energy is reconverted into electric energy and the rest remains stored in the magnetic circuit containing the magnetic material. If the magnetomotive force is now increased in the opposite direction the associated changes in flux density will be represented typically by the curve (Fig. 20) connecting B_p with $-B_m$. A reduction of magnetomotive force to zero, reversal, and increase in the initial direction again will bring the flux density back to $+B_m$ along the lower curve (Fig. 20) connecting $-B_m$ with $+B_m$. The complete sequence of changes in B corresponding to nI represented in Fig. 20 is called the hysteresis cycle of the magnetic material and its graphical representation is called a hysteresis loop. During the hysteresis cycle the magnetic energy not reconverted into electric energy is converted into heat. The energy lost in this manner in a magnetic material subjected to a reversing magnetomotive force (by applying the principle stated in 43 to a cubic centimeter of the material) is given by $w = \int_{-B_m}^{+B_m} \frac{nI\,dB}{10}$ ergs per cubic centimeter, or the area enclosed by the hysteresis loop divided by 10.

Magnetic Poles. The surface separating any two materials of different permeabilities and permeated by a magnetic flux is the apparent seat of a force acting upon magnetic materials and such a surface is called a magnetic pole. The surface through which the flux leaves a magnetic material of higher permeability is called a North pole and the surface through which the flux enters a magnetic material of higher permeability is called a South pole.

Force between Two Magnetic Poles. When two magnetic materials are separated by a material of permeability (μ) and the flux density normal to the adjacent surfaces of the magnetic materials is B gausses the magnetic energy stored within the separating material, from 46, is $\frac{B^2}{8\pi\mu}$ ergs per cubic centimeter. If the area of each of the adjacent surfaces of the magnetic materials (Fig. 21) is A square centimeters and the thickness of the separating material is increased by dx centimeters the volume of the separating material is increased by $A\,dx$ cubic centimeters. The energy of the magnetic circuit is therefore increased by $\frac{B^2A\,dx}{8\pi\mu}$ when the distance between the adjacent surfaces is increased by dx centimeters. If F is the force in dynes between the poles located on the adjacent surfaces the work done in moving one of the poles a distance of dx centimeters is $F\,dx$ ergs. Hence $F\,dx = \frac{B^2A\,dx}{8\pi\mu}$ and the force between the two poles is given by

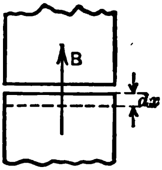


FIG. 21

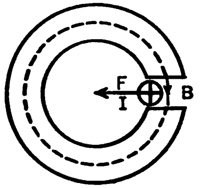


FIG. 22

51 $$F = \frac{B^2A}{8\pi\mu} \text{ dynes.}$$

In Fig. 22, the force acting on the wire, from 36, is given by $dF = \frac{BI\,ds\sin\alpha}{10}$ dynes. If B is produced by I, then I may be expressed as a function of B and the force acting on the wire is proportional to the square of the flux density. It will therefore be noted that the force between a wire conducting a current and a magnetic material, in which the flux is established by, and is proportional to the same current, is also proportional to the square of the flux density established in the material.

General Principle of Force Action in a Magnetic Field. The force acting upon a current or a magnetic material placed in a magnetic field is in such a direction as to increase the flux linking the current or permeating the magnetic material. Applications of this principle may be summarized as follows:

Case I. A wire conducting a current.

- *a.* A bent wire will tend to straighten.
- *b.* A circular wire will tend to increase its diameter.
- *c.* A coil of two or more turns will tend to become more compact

Case II. Two wires conducting currents.

- *a.* Force of attraction between two currents flowing in the same direction.
- *b.* Force of repulsion between two currents flowing in opposite direction.
- *c.* Two currents making an angle with each other will tend to twist into the same plane with their currents flowing in the same direction.

Case III. A wire conducting a current, and a material.

- *a.* A magnetic material will be attracted by the current to a position where the flux density due to the current is a maximum.
- *b.* A diamagnetic material will be repelled.
- *c.* A permanent magnet will tend to turn into a position such that the flux permeating it due to the wire and itself will be a maximum.

Case IV. A permanent magnet and a material.

- *a.* A magnetic material will be attracted toward the nearer pole of the permanent magnet.
- *b.* Several magnetic materials will be attracted as in (*a*) and also by each other, forming several chains.
- *c.* Each pole of another permanent magnet will be attracted by the opposite pole and repelled by the like pole of the original permanent magnet.

CHAPTER III

ELECTROMAGNETIC INDUCTION

Electromotive Force generated or induced in a Material. It was demonstrated on page 19 that the work done in moving a wire ds centimeters in length conducting a current of I amperes through a magnetic field of B gausses flux density a distance of dx centimeters to the left, as shown in Fig. 23, is given by $dw = \frac{Bi\,ds\,dx}{10}$ ergs. The work done upon a wire of finite length (s centimeters) is then given by $dw = \frac{Bis\,dx}{10}$ ergs. The power, or rate at which work is done on the wire in moving it dx centimeters in dt seconds, is therefore given by $P = \frac{Bis\,dx}{10\,dt}$ ergs per second or $\frac{Bisv}{10^8}$ watts, where v is the velocity of the wire in centimeters per second.

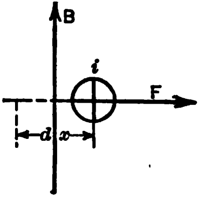


FIG. 23

The mechanical energy thus expended in moving the wire is converted into electric energy of the same amount which appears within the wire and its associated circuit. Hence the electric power developed by the motion of the wire in the magnetic field is also given by $P = \frac{Bisv}{10^8}$ watts and, since the process is reversible, the electromotive force generated in the wire, from 1, is given by $e = \frac{Bisv}{i\,10^8}$ or $\frac{Bsv}{10^8}$ volts. If N conductors connected in series are moved simultaneously through a magnetic field under the conditions indicated in Fig. 23 and the e.m.f. generated in each conductor acts in the same direction in the series connection the total e.m.f. is given by

$$e = \frac{NBvs}{10^8} \text{ volts.} \tag{52}$$

Since each moving conductor (Fig. 23) is a source of electric power the direction of the e.m.f. (defined on page 3) is the same as that of the current. The relative directions of the motion (M), the flux density (B), and the electromotive force (E) are shown in Fig. 24. If the directions of the motion and the flux density are indicated respectively by the thumb and forefinger of the right hand the direction of the e.m.f. is indicated by the middle finger.

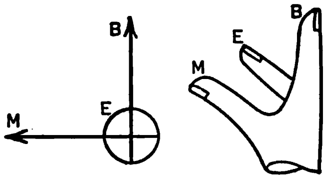


FIG. 24

Since a magnetic field is permeated by continuous tubes of flux the rate of change of the flux $\left(\frac{d\phi}{dt}\right)$ linking the N wires when moving at a velocity of v centimeters per second is Bvs maxwells per second and the electromotive force generated in the N moving conductors or induced in the N conductors when the flux linking them changes at a rate of $\frac{d\phi}{dt}$ maxwells per second is given by

53 $$e = \frac{N}{10^8}\frac{d\phi}{dt} \text{ volts.}$$

The direction of a generated or induced electromotive force may be determined by the right hand rule or by the principle that the electromotive force produced by a changing linkage flux acts in such a direction as to oppose (by the current it tends to establish) the change of linkage flux. The direction of the electromotive force is therefore given by the direction of the current (Fig. 13) which will establish a flux in such a direction as to maintain the original magnitude and direction of the linkage flux.

Electromotive Force induced in a Wire by a Change of Current in the Same Wire. When the flux linking N turns of wire is directly proportional to the current flowing in the wire, $\phi = Ki$ and $\frac{d\phi}{dt} = K\frac{di}{dt}$. The electromotive force induced in the N turns of wire when the current flowing in the wire changes at a rate $\left(\frac{di}{dt}\text{ amperes per second}\right)$, from 53, is given by $e = \frac{NK}{10^8}\frac{di}{dt}$ volts. Since N, K and 10^8 are constants (N and K are constants of the circuit), a single constant (L) may be substituted for $\frac{NK}{10^8}$ and this constant is called the self-inductance of the circuit, measured in henries. In any circuit of L henries self-inductance, the electromotive force induced in the circuit when its current changes at a rate of $\frac{di}{dt}$ amperes per second is therefore given by

54 $$e = L\frac{di}{dt} \text{ volts.}$$

The direction of the self-induced electromotive force, in accordance with the general principle stated under 53, will be such as to oppose the change in linkage flux or the change in current. It will be noted that the self-inductance of a circuit produces a steadying effect upon the current and is analogous to the mass property of matter which steadies the motion of matter through space.

The Self-inductance of a Wire. The self-inductance of any formation of wire may be determined from the ratio of the flux linking the wire to the current flowing in the wire which produces the linkage flux. This ratio (K) multiplied by the number of turns (N) linked by the flux and divided by 10^8 gives the self-inductance of the wire. The value of K may be determined by an integration of 37; the self-inductance of two parallel wires, for example, conducting the same current in opposite directions (the transmission line), determined by integrating the flux permeating the space between the wires and the wires themselves, is given by

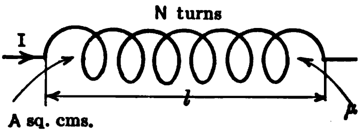


FIG. 25

55 $$L = \left[0.0805 + 0.741 \log \frac{D}{r}\right] 10^{-3} \text{ henries per wire per mile.}$$

D is the distance between the centers of the wires and r is the radius of each wire, D and r being measured by any unit of length provided the same unit is used for both.

In simpler cases, K may be determined by substituting the constants of the circuit in 47. In a long solenoid of wire (Fig. 25), from 47, $\Phi = \dfrac{1.26\,NI}{\dfrac{l}{\mu A}} = \left(\dfrac{1.26\,N\mu A}{l}\right) I$ and $K = \dfrac{1.26\,N\mu A}{l}$.

Hence

56 $$L = \frac{1.20\,N^2\mu A}{l \times 10^8} \text{ henries.}$$

It will be noted that the self-inductance of any formation of wire depends upon its geometrical constants and the nature of the surrounding medium. The self-inductance is independent of the current if the wire is surrounded by a medium of constant permeability, but if the permeability of the medium is not constant, the self-inductance of the wire is a variable depending upon the magnitude of the current. The self-inductance of any formation of wire linked by Φ maxwells when conducting I amperes, substituting $\frac{\Phi}{I}$ for K in $L = \frac{NK}{10^8}$, is given by

57 $$L = \frac{N\Phi}{10^8 I} \text{ henries.}$$

Electromotive Force induced in a Wire by a Change of Current in Another Wire. In Fig. 26 if the flux (ϕ_2) linking coil (2) due to a current (i_1) in coil (1) is directly proportional to i_1, or $\phi_2 = K'i_1$, then $\frac{d\Phi_2}{dt} = K'\frac{di_1}{dt}$ and from 53, $e_2 = \frac{N_2K'}{10^8}\frac{di_1}{dt}$. Since N_2, K' and 10^8 are constants (N_2 is a constant of coil (2) and K' is a constant involving the geometrical constants of both coils and the nature of the surrounding medium) a single constant (M) may be substituted for $\frac{N_2K'}{10^8}$ and this constant is called the mutual-inductance of the two coils, measured in henries. In two circuits of M henries mutual-inductance the electromotive force induced in one circuit when the current in the other circuit changes at a rate of $\frac{di_1}{dt}$ amperes per second is given by

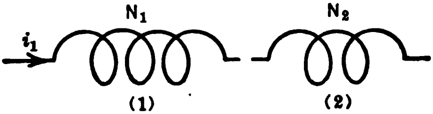


FIG. 26

58 $$e_2 = M\frac{di_1}{dt} \text{ volts.}$$

The Mutual-inductance of Two Wires. The mutual-inductance of two formations of wire may be determined from the ratio of the flux linking wire (2), due to a current (i_1) in wire (1), to the current in wire (1). This ratio (K') multiplied by the number of turns in wire (2), linked by the flux established by the current in wire (1), and divided by 10^8 gives the mutual-inductance of the two wires. The value of K' may be determined by an integration of 37 or, in simpler cases, by substituting the constants of wire (1) in 47 and determining the amount of flux due to the current in wire (1) that links wire (2). In the case of two long solenoids wound together on the same core (Fig. 27) the flux (Φ_1) due to the current (I_1) flowing in solenoid (1), from 47, is given by

$$\Phi_1 = \frac{1.26\, N_1 I_1}{\dfrac{l_1}{\mu_1 A_1}} \quad \text{or} \quad \left(\frac{1.26\, N_1 \mu_1 A_1}{l_1}\right) I_1.$$

Since the same flux links solenoid (2),

$$K' = \frac{1.26\, N_1 \mu_1 A_1}{l_1}$$

and

59 $$M = \frac{1.26\, N_1 N_2 \mu_1 A_1}{l_1 \times 10^8} \text{ henries.}$$

Self-inductance of a Series Connection. Given the self-inductances (L_1, L_2, L_3. etc.) of the respective parts of a series connection in which there is no mutual-inductance between the respective parts the resultant self-inductance of the series connection is given by

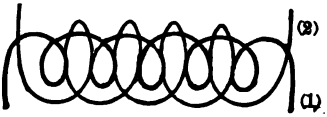


FIG. 27

60 $$L = L_1 + L_2 + L_3 + \text{etc. henries.}$$

Given the mutual-inductance (M) of two parts of a series connection and the self-inductances (L_1 and L_2) of the two parts respectively, the resultant self-inductance of the series connection is given by

61 $$L = L_1 \pm 2\,M + L_2 \text{ henries.}$$

The sign before $2\,M$ is + when the mutual fluxes are in conjunction and is − when the mutual fluxes are in opposition. Since the mutual-inductance of two parts of a series connection linked by the same flux, from 59, equals kN_1N_2 and the self-inductance of each part, from 56, equals kN_1^2 and kN_2^2 respectively, then

$$N_1 = \sqrt{\frac{L_1}{k}}, \quad N_2 = \sqrt{\frac{L_2}{k}} \quad \text{and} \quad M = k\sqrt{\frac{L_1}{k}}\sqrt{\frac{L_2}{k}}$$

or

62 $$M = \sqrt{L_1L_2} \text{ henries.}$$

The resultant self-inductance of a parallel connection depends upon the division of the current among the parts of the parallel connection and cannot be stated in terms of the respective self-inductances of its constituent parts. This subject is discussed further on page 85.

Energy required to establish a Current. When the flux (Φ) linking the N turns of an electric circuit is directly proportional to the current (I) flowing in the circuit, the energy required to establish the current, from 45, is given by $W = \frac{N\Phi I}{20}$ ergs or $\frac{N\Phi I}{2 \times 10^8}$ joules. Since $\Phi = KI$, $W = \frac{NKI^2}{2 \times 10^8}$ and, substituting L for $\frac{NK}{10^8}$,

63 $$W = \tfrac{1}{2} LI^2 \text{ joules.}$$

Potential between Two Points in a Series Connection conducting a Variable Current. When the current flowing between the two points a and b in Fig. 28 is constant the potential between a and b, from 6, is given by $V_{ab} = +E - IR$. If the current flowing from a to b is variable and the series connection between a and b possesses self-inductance, the potential between a and b, combining 6 and 54, is given by

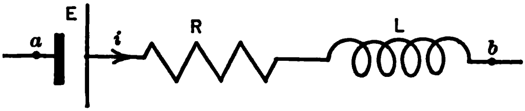


Fig. 28

64 $$v_{ab} = +E - iR - L\frac{di}{dt}$$

where E, i and R are defined as in 6, L is the self-inductance between a and b, and $\frac{di}{dt}$ is the rate of change of the current; $\frac{di}{dt}$ is positive when the current flowing from *a* to *b* is increasing and is negative when the current flowing from *a* to *b* is decreasing.

Current flowing at Any Instant in a Series Circuit. In a series circuit containing a constant electromotive force (E), resistance (R) and self-inductance (L), as in Fig. 29, the potential between the two points *a* and *b* becomes zero when the switch is closed and, from 64, $+E - iR - L\frac{di}{dt} = 0$. This is a linear differential equation of the first order and first degree, and its solution gives

65
$$i_t = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right) + I\epsilon^{-\frac{Rt}{L}} \text{ amperes}$$

where i_t is the current flowing in the circuit t seconds after the switch is closed, ϵ is the base of the natural system of logarithms (2.713), and I is the current flowing in the circuit, in the direction of the ultimate current, at time ($t = 0$).

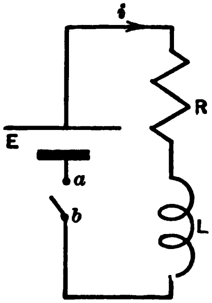


FIG. 29

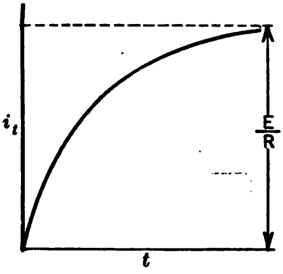


FIG. 30

If the current flowing in the circuit when the switch is closed ($t = 0$) is zero, 65 reduces to

66
$$i_t = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right) \text{ amperes.}$$

The curve represented by 66 is plotted in Fig. 30 and indicates that the current in such a circuit increases from zero to its ultimate value $\left(\frac{E}{R}\right)$ at a diminishing rate. The time required for the current to reach its approximate ultimate value (an infinite time is required to reach its actual ultimate value) is proportional to the ratio $\left(\frac{L}{R}\right)$, called the time constant of the circuit. When $\frac{L}{R}$ is small, the current increases rapidly and when $\frac{L}{R}$ is large the current increases slowly.

If the switch in Fig. 30 is suddenly thrown to a', as in Fig. 31, the electromotive force (E) in 65 is eliminated from the circuit and 65 reduces to

$$i_t = I\epsilon^{-\frac{Rt}{L}} \text{ amperes,} \tag{67}$$

where i_t is the current flowing in the circuit t seconds after the switch is thrown to a', R equals $R_1 + R_2$, and I is the current flowing in R_1 and L at time ($t = 0$).

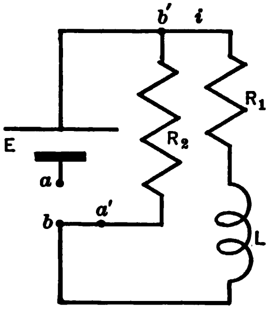


Fig. 31

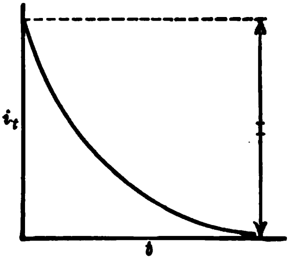


Fig. 32

The curve represented by 67 is plotted in Fig. 32 and indicates that the current diminishes from its initial value to zero at a diminishing rate. The time required for the current to reach zero approximately (an infinite time is required to reach zero exactly) is proportional to the ratio $\left(\frac{L}{R_1 + R_2}\right)$, called as before the time constant of the circuit. When $\frac{L}{R_1 + R_2}$ is small, the current decreases rapidly and when $\frac{L}{R_1 + R_2}$ is large the current decreases slowly.

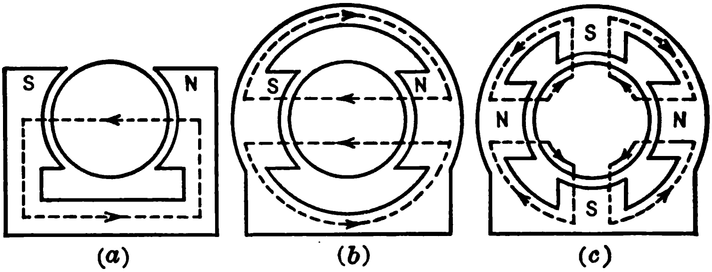
Since the initial value of the current flowing through R_1, L, and R_2 is the same as that flowing through R_1 and L just before the e.m.f. (E) was eliminated from the circuit, the initial potential between a' and b' ($v_{a'b'} = -IR_2$) may be many times greater than the original electromotive force impressed upon the circuit.

CHAPTER IV

THE DIRECT CURRENT DYNAMO

Application of the Dynamo. A dynamo is designed for the continuous conversion of mechanical into electric energy or the reverse. When mechanical energy is converted into electric energy the dynamo is called a generator and when electric energy is converted into mechanical energy the dynamo is called a motor. The power delivered from a dynamo is called its load. Generators are usually driven at constant speed at all loads while motors may vary in speed with the load, the character of the speed variation in any case depending upon the type of motor.

Construction of the Dynamo. The principal features of construction of direct current dynamos intended for either generator or motor action are essentially the same. All direct current dynamos contain a stationary structure called the frame and a rotating structure called the armature. The frame includes the base, the field core and the members which support the armature bearings. The field core, which forms part of the magnetic circuit of the dynamo, may contain a single path as in Fig. 33 (*a*) or several paths as in 33 (*b*) and 33 (*c*). The parts of the field core adjacent to the armature are called the field poles and the remainder of the field core is called the field yoke. Dynamos may be constructed with field cores containing any multiple of two field



(*a*) (*b*) (*c*)

FIG. 33

poles; examples of bipolar construction are indicated in Figs. 33 (*a*) and 33 (*b*), and an example of multipolar construction (a four pole field core) is indicated in Fig. 33 (*c*). The field core may consist of a single steel casting which includes the base of the dynamo or may consist of several parts bolted together. The poles are often constructed of laminated sheet iron and in some cases the entire field core is of laminated construction. The armature core is built up of laminated sheet iron stampings keyed to a steel shaft or a cast steel spider attached to the shaft.

Electromotive Force generated in an Armature Conductor. The armature conductors of a dynamo are usually embedded in parallel slots on the surface of the armature core. The distribution of the flux density normal to the surface of the armature core under two poles may be represented typically by the curve (*B*) shown in Fig. 34. Since the electromotive force generated in a moving conductor, from 52, is proportional to the flux density normal to its plane of motion, the electromotive force generated at any instant in each armature conductor as it moves from *a* to *a′* must therefore vary in the same manner as the flux density. The electromotive force (curve *e*) generated in each moving conductor is therefore zero at *a*, a positive maximum at *b*, zero at *c*, a negative maximum at *d*, zero at *a′*, etc. An electromotive force which varies in this manner is called an alternating electromotive force and is discussed in detail on page 73.

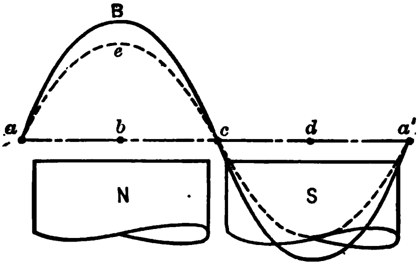


FIG. 34

The Bipolar Drum Winding. To impress a direct electromotive force upon a circuit connected to such a dynamo the armature conductors must be connected to each other and to the external circuit so that the electromotive forces of all conductors in the armature which are connected in series will act in the same direction throughout that connection at any instant and the positive terminal of each of these series connections must be connected continuously to the positive terminal of the external circuit.

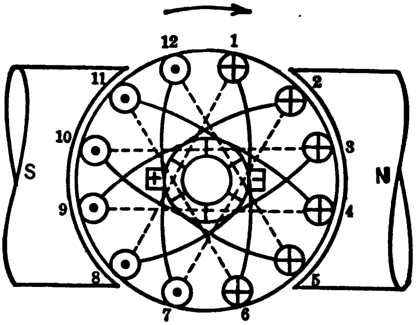


FIG. 35

The manner in which these requirements are satisfied in the bipolar dynamo is illustrated in Fig. 35. The various conductors on the armature are connected front and back to form an endless winding, called a drum winding. Each front connection is attached to a segment in a multi-segment ring, called a commutator, and the connections to the external circuit are made through two brushes which rest upon the commutator at diametrically opposite points. The current flowing into the winding at the negative brush divides at the point of brush contact, flows through two paths in the armature winding and leaves the armature at the positive brush.

For the position of the armature shown in Fig. 35, the armature conductors are connected to the external circuit as shown diagrammatically in Fig. 36. The electromotive force generated in each conductor at this instant is shown in magnitude and direction in Fig. 37. The aggregates of the electromotive forces generated in each path for any position of the armature are the same but are not constant. If the flux density normal to the surface of the armature is distributed sinusoidally and the width of the brushes is assumed negligible the electromotive force generated in each path will vary from $E \cos \frac{\alpha}{2}$ to E to $E \cos \frac{\alpha}{2}$ as the brush passes over each commutator segment, where α is the angle subtended by each commutator segment. The electromotive force generated between the brushes will be unidirectional in such a dynamo but will vary slightly as the armature rotates, the magnitude of the variation being less as the number of commutator segments is increased.

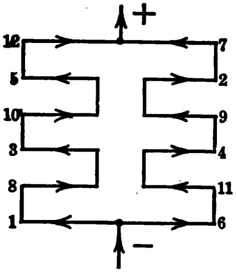


FIG. 36

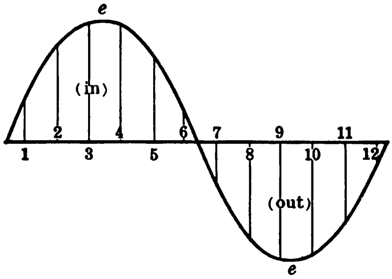
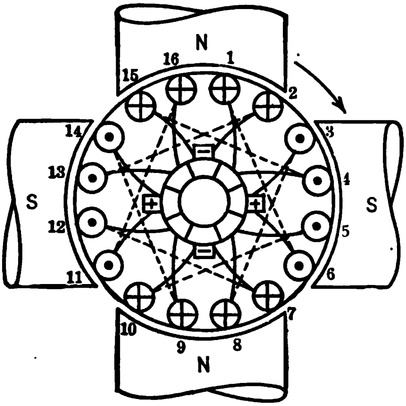


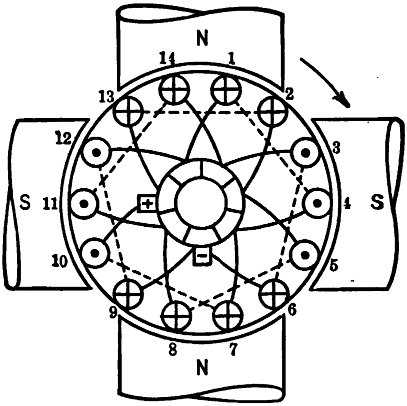
FIG. 37

The Multipolar Drum Winding. The size of the armature conductors in any dynamo depends upon the maximum current that the dynamo is expected to carry continuously. The number of armature conductors depends upon the speed of the armature and the electromotive force to be generated in the armature winding. When the space requirements of the armature conductors



4 pole lap winding

FIG. 38



4 pole wave winding

FIG. 39

becomes large by reason of their size or number the requisite diameter of the armature may for many reasons suggest the use of a multipolar field core. Two types of drum winding may be employed in connection with a multipolar field core, a lap winding (shown in Fig. 38) and a wave winding (shown in Fig. 39).

In the lap winding the conductors are joined by alternate forward and backward connections while in the wave winding the conductors are joined by successive forward connections. Two or more windings similar to those illustrated in Figs. 38 and 39 and placed side by side may be employed to increase the number of paths through the armature. The winding is said to be simplex, duplex or triplex when it contains one, two or three such windings. These windings may be connected in series (called singly-reëntrant) or may be independent of each other (called doubly-reëntrant, triply-reëntrant, etc.)

The number of paths between brushes and the number of brushes required in a dynamo having p poles is given as follows for the various types of drum winding.

	Lap		Wave	
	Paths	Brushes	Paths	Brushes
Simplex	p	p	2	2
Duplex	$2p$	p	4	2
Triplex	$3p$	p	6	2

While more than two brushes may be employed theoretically in connection with the multipolar wave winding practical conditions favor the use of two brushes. Brushes covering two or more commutator segments are used in connection with multiplex windings.

Winding Formulas for Simplex Drum Windings. Simplex windings are used almost exclusively in most dynamos and the rules for such windings only are therefore given in terms of the following symbols.

Number of conductors in the winding, Z; number of poles, p; forward pitch (number of conductors spanning an end connection not including the conductor from which the connection starts),

y_f; backward pitch (counted in the same manner as y_f), y_b; average pitch $\left(\frac{y_f + y_b}{2}\right)$, y_a; any integral constant, k.

Lap or wave winding; Z must be even and both y_f and y_b must be odd. Lap winding; $y_a = \frac{Z}{p}$ approximately and must be even. $y_f - y_b = 2\,k$. Wave winding; $y_a = \frac{Z \pm 2\,k}{p}$ and may be odd or even. When y_a is odd $y_f = y_b$, and when y_a is even $y_f - y_b = 2\,k$.

Commutator and Brush Construction. A commutator consists of a ring of copper segments clamped between two cast-iron spiders, the copper segments being insulated from each other and from the spiders by reconstructed mica. The number of commutator segments equals the number of armature conductors divided by an integer. Although it is desirable to employ as large a number of commutator segments as possible to reduce the pulsations of the generated e.m.f. between brushes a large number of commutator segments implies a large diameter of the commutator and the number of segments may be limited by the speed of the brush over the surface of the commutator at which satisfactory commutation may be obtained.

Brushes, usually made of carbon, are supported by brush holders which maintain sufficient pressure of the brushes on the commutator to obtain proper contact without excessive wear. The brush holders are attached to a brush yoke by insulated studs. The brush yoke is bolted to the frame of the dynamo and in many cases is adjustable as to position so that the brushes may be moved together through an angle around the commutator to improve commutation.

Electromotive Force generated in a Drum Wound Armature. If S equals the rotational speed of an armature in revolutions per minute the time required for one revolution is $\frac{60}{S}$ seconds and the time required for one conductor to pass through the space between two consecutive interpolar planes is $\frac{60}{pS}$ seconds. If Φ is the total flux entering or leaving each pole the average rate at which each conductor cuts magnetic flux is $\frac{\Phi}{\frac{60}{pS}}$ or $\frac{\Phi p S}{60}$ maxwells per second.

The average electromotive force generated in each conductor from 53, is then given by $E_{av.} = \frac{\Phi p S}{6 \times 10^9}$ volts. If there are Z conductors on the armature and m paths between terminals there will be $\frac{Z}{m}$ conductors connected in series in each path. Hence the electromotive force generated in each path, or the electromotive force generated in the armature, is given by

$$E_a = \frac{Z \Phi p S}{6\, m \times 10^9} \text{ volts.} \tag{68}$$

Dynamo Excitation. The magnetic flux which traverses the armature core, the air gaps and the field core may be produced by a permanently magnetized field core or by passing an electric current through coils wound on the field poles. The source of the magnetic flux in either case is called the excitation of the dynamo. Excitation by permanent magnetism is limited (1) by the low flux density attained in this manner, (2) by its inflexibility of adjustment, and (3) by the tendency of the flux density to decrease with age, vibration and temperature changes. Permanent excitation is used extensively, however, in small dynamos (called magnetos) in connection with gas engine ignition systems and in certain types of speed indicators. When the magnetic flux is produced by coils of wire carrying electric current the number of ampere turns required to produce a given flux may be determined from 47 or 50.

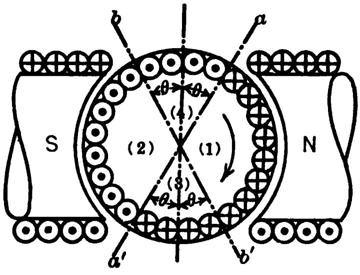


FIG. 40

Armature Reaction. In any dynamo operated under load the brushes (to improve commutation as indicated later) may be located at an angle (θ) from the interpolar plane as shown in Fig. 40 which represents the conditions existing under the circumstances in a generator. Since the current in each conductor to the right of the plane (aa') flows downward and the current in each conductor to the left of the plane (aa') flows upward it is evident that the armature conductors when carrying current constitute a source of magnetomotive force which acts from a' toward a.

This magnetomotive force, called the armature reaction, may be resolved into two components. The magnetomotive force of the ampere conductors subtending sectors (1) and (2) is cross-magnetizing and the magnetomotive force of the ampere conductors subtending sectors (3) and (4) is demagnetizing. The cross-magnetizing magnetomotive force acting together with the field magnetomotive force distorts the path of the magnetic flux through the armature, the amount of distortion depending upon the load on the dynamo. The demagnetizing magnetomotive force opposes the field magnetomotive force and decreases the field flux, the amount of reduction in flux depending upon the load on the dynamo.

The demagnetizing ampere turns in the armature of any dynamo are given by

69 $$N_a I_a = \frac{Z I_a \theta}{360\, m} \text{ ampere turns per pole,}$$

where Z is the number of armature conductors, I_a is the total armature current, θ is the angle between the brush plane and the interpolar plane, and m is the number of armature paths between terminals. The number of ampere turns required in the field winding per pole to produce a flux (Φ) in the magnetic circuit when the armature current is I_a amperes is then given approximately, from 69 and 47, by

70 $$N_f I_f = N_a I_a + \frac{\Phi}{2.52}\left[\mathcal{R}_{\text{arm. core}} + \mathcal{R}_{\text{air gap}} + \mathcal{R}_{\text{field}}\right]$$

ampere turns per pole.

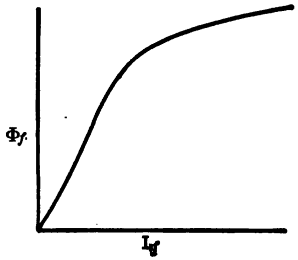


FIG. 41

Self-excitation. The current established in the field winding of a dynamo may be due to an external electromotive force (separate excitation) or to the electromotive force generated in the armature (self-excitation). In either case (neglecting the effect of armature reaction) the field flux (Φ_f) corresponding to any value of field current (I_f) will be represented by a curve as shown in Fig. 41. Since the generated electromotive force at any stated speed is directly proportional to the flux per pole, a plot of the generated electromotive force (E_a) and the field current (I_f) with the armature speed held constant will be of the same form as that shown in Fig. 41.

Corresponding values of generated e.m.f. and field current are plotted in Fig. 42 for a dynamo operated at constant speed. If the excitation of a dynamo is provided by an external source of e.m.f. (E_b) and the e.m.f. generated in the armature of the dynamo is of the same magnitude (E_a), it is evident (neglecting the resistances of the external source and the resistance of the armature) that the armature will continue to generate the same e.m.f. if the field switch (Fig. 43) is thrown to the right.

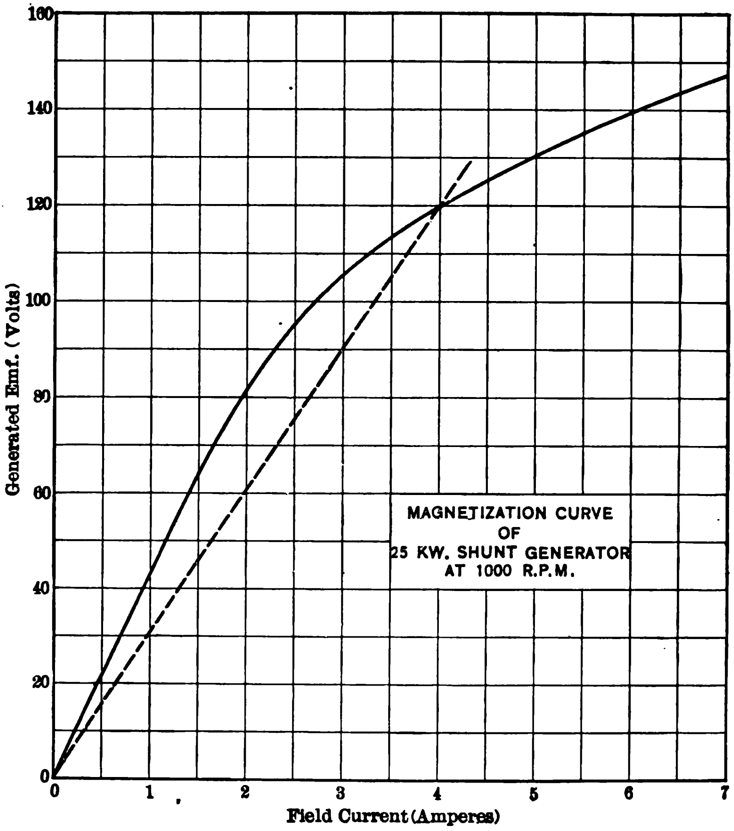


FIG. 42

The Shunt Generator. A dynamo operating as a generator and self-excited by a field winding connected to the armature terminals as shown in Fig. 44 is called a shunt generator. It is usually unnecessary to employ an external source of excitation to obtain the initial generation of e.m.f. in the armature of the shunt generator since the magnetic circuit will in general be traversed by a small amount of flux due to the permanent magnetism of the field core. A small e.m.f. will therefore be generated in the armature when it is brought up to speed and this e.m.f. will establish a small current in the field winding. If the magnetomotive force due to this field current assists the initial permanent magnetomotive force the resultant flux will be increased. The generated e.m.f., field current and field flux will then increase simultaneously as shown in Fig. 45 and the generator is said to "build up."

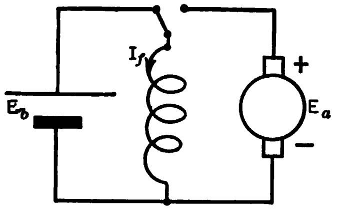


FIG. 43

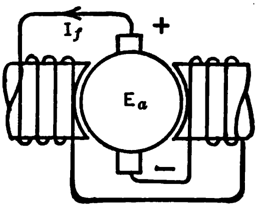


FIG. 44

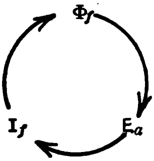


FIG. 45

The generated e.m.f. (E_a) will continue to increase so long as E_a is greater than the product of the field current (I_f) and the field resistance (R_f), the armature resistance being assumed negligible compared with the field resistance. To find the point on the curve in Fig. 42 at which the generated e.m.f. ceases to increase, a line may be drawn from the origin making an angle with the horizontal axis such that any ordinate to this line measured in volts divided by the corresponding abscissa measured in amperes equals the resistance of the shunt field. The point of intersection of this line with the magnetization curve indicates the ultimate value of the generated e.m.f. since this value of E_a equals I_fR and is also the E_a corresponding to the same I_f. If the resistance line and the magnetization curve do not intersect, the generator will not "build up." If a line is drawn from the origin tangent to the magnetization curve the ratio of any ordinate to this line to the corresponding abscissa is called the critical resistance of the shunt field.

The armature current of a shunt generator operated at no load equals the field current, but when a load is connected to the generator terminals the armature current equals the sum of the field current and the line current as shown in Fig. 46. The application of load to the shunt generator is accompanied by a decrease in terminal potential for the following reasons:

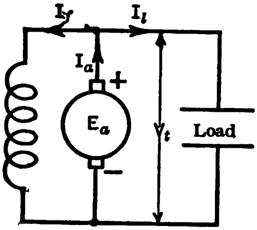


FIG. 46

(1) The terminal potential of a shunt generator, from 6, is given by $V_t = E_a - I_aR_a$. If E_a is first assumed to remain constant it is evident that V_t must decrease as I_a increases.

(2) The field current of a shunt generator is given by $I_f = \frac{V_t}{R_f}$. Since V_t decreases with an increase of load as indicated in (1) I_f, and (consequently) Φ and E_a must decrease.

(3) An increase in armature current being associated with an increase in load the demagnetizing m.m.f. of the armature must increase with the load and both Φ and E_a must decrease.

A plot of corresponding values of E_a and I_l (called the internal characteristic) and corresponding values of V_t and I_l (called the external characteristic) for a shunt generator under increasing load are shown in Fig. 47. In a properly designed generator with low armature resistance and low demagnetizing m.m.f. the change of terminal potential from no load to full load is usually less than five per cent. The terminal potential may be held constant for any load or made higher than at no load by manual or automatic adjustment of the shunt field resistance.

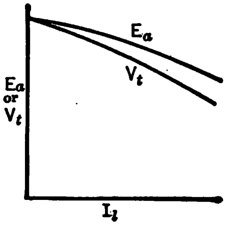


FIG. 47

The Series Generator. When the excitation of a dynamo operated as a generator is furnished by the line current flowing through field coils connected in series with the load and the armature, as shown in Fig. 48, the dynamo is called a series generator.

The generated e.m.f. at no load will then be due to the permanent magnetism of the field core and as the line current (I_l) increases, since $I_l = I_s = I_a$, the generated e.m.f. (E_a) will increase as shown in Fig. 49. The terminal potential (V_t) for any value of the load will be less than E_a by an amount equal to $I_aR_a + I_sR_s$ as shown in Fig. 49. The upper curve in Fig. 49 is the internal characteristic and the lower curve the external characteristic of the series generator. Such a generator is obviously not adapted to a constant potential system operating under variable load. Although employed to some extent in connection with series street lighting systems operated at constant current and variable potential the series generator at present has very few applications.

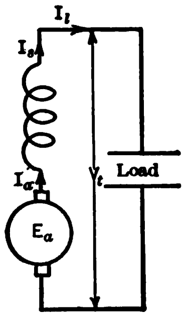


FIG. 48

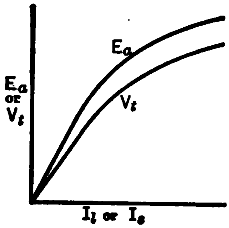


FIG. 49

The Compound Generator. A generator equipped with both shunt and series field windings and called a compound generator (Fig. 50) may be employed with advantage in many instances. The generator is called a short-shunt compound generator when the shunt winding is connected across the brushes and is called a long-shunt compound generator when the shunt winding is connected across the line terminals. Since the terminal potential of a shunt generator decreases with an increase of load and the terminal potential of a series generator increases with an increase of load, by combining the two windings in various proportions the terminal potential may be made to increase (over-compounded), remain constant (flat-compounded) or decrease (under-compounded) as the load increases as shown in Fig. 51. The degree of compounding may be regulated by adjustment of the shunt field resistance or by adjustment of a resistance connected in parallel with the series field winding.

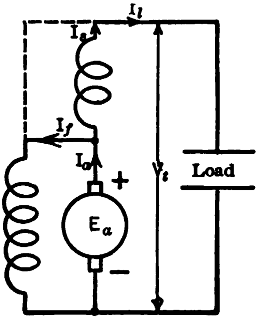


FIG. 50

Over-compounded generators are employed in connection with loads located at some distance from the generator and provide compensation for the potential drop in the transmission line. Flat-compounded generators provide constant potential at the terminals at any load. Under-compounded generators are employed in general when compound generators are connected in parallel since the inherent distribution of load between such generators is more uniform if each generator possesses a drooping external characteristic.

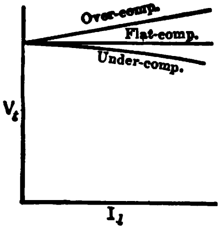


FIG. 51

Parallel Operation of Electric Generators. Power plants are in general equipped with two or more generators. In many cases a single generator could not be obtained which would supply the entire load on the plant. The use of a single generator would, moreover, reduce the reliability of operation and no opportunity would be offered for making repairs without shutting down. Generators should be selected as to number and size in any case to operate at the least annual cost under the present and future load curve of the plant. The number of generators installed in a plant may also be increased to meet the demands of an increased load on the plant due to extensions or increased production. Unless special conditions favor the separate operation of each generator in connection with certain parts of the load, all generators will be connected in parallel to the main bus bars and each generator will supply its share of the entire load on the plant.

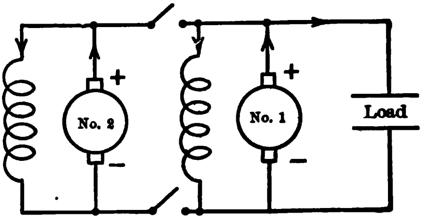


FIG. 52

Parallel Operation of Shunt Generators. If a shunt generator, No. 1 in Fig. 52 for example, is supplying power to a load and another shunt generator, No. 2, is to be connected in parallel with it the terminal potentials of the two generators must first be adjusted to the same magnitude and polarity. The switch connecting the two generators may then be closed and the load may be distributed between the two generators by adjustment of the shunt field resistances or the armature speeds. The inherent distribution of load between the two generators will depend upon their external characteristics. If the external characteristics are not of similar form each generator will supply the same proportion of its rated capacity to the load only at the terminal potential given by the intersection of the characteristics as shown in Fig. 53. The characteristics must be identical in form if the load is to be distributed inherently between the two generators at all loads in the same proportion of the rated capacity of each generator.

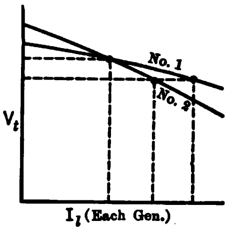


FIG. 53

Parallel Operation of Compound Generators. If a compound generator, No. 1 in Fig. 54, is supplying power to a load and another compound generator, No. 2, is to be connected in parallel with it the terminal potentials of the two generators must first be adjusted to the same magnitude and polarity. If switches S_2 and S_3 are now closed the two generators may operate satisfactorily in parallel but the operation with such connections is generally unstable. If the potential between *c* and *d* should drop for any reason below the potential between *a* and *b*, current may flow through the series field of No. 2 in the opposite direction to the normal series field current so that the series field magnetomotive force will oppose that of the shunt field. This reversal of current in the series field of No. 2 will then cause a cumulative reduction

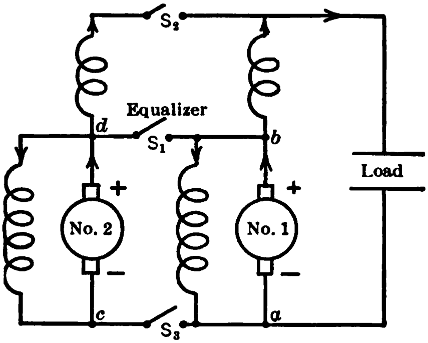


FIG. 54

in the generated e.m.f. of No. 2 until No. 2 may finally be driven as a motor by No. 1. This action may overload No. 1 to the extent that its protective devices will disconnect it from the circuit. If No. 2 is of insufficient capacity to supply the load it will also be disconnected from the load and the system will be without a source of power.

The instability of operation just described may be eliminated by joining *b* and *d* in Fig. 54 with a low resistance connection called the equalizer. With such a connection the potential between *a* and *b* must be the same as that between *c* and *d*, and the reversal of either series field current is made impossible. The inherent distribution of load between two compound generators connected in parallel will depend upon their external characteristics and the relative resistances of their series field windings. For proportionate distribution of load between the two generators for any general load on the system the external characteristics should be identical in form and $\frac{(I_l)_1}{(I_l)_2}$ must equal $\frac{(R_s)_2}{(R_s)_1}$, where I_l and R_s are the full load line current and series field resistance respectively of each generator. The distribution of load between the two generators may be regulated by adjustment of the resistance of the shunt fields, series fields, and sometimes the equalizer, or by adjustment of the armature speeds.

Principles of Commutation. The currents flowing in the various parts of a generator armature adjacent to one of the brushes are shown in Fig. 55. I_x and I_y are the two currents which flow through two paths in the armature toward the positive brush. When the brush is centered on segment No. 1 of the commutator,

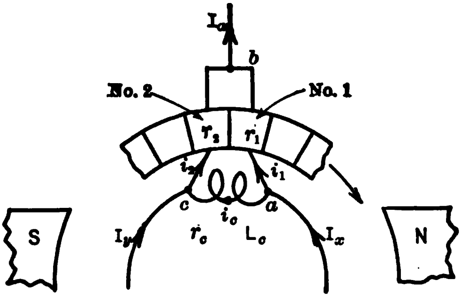


FIG. 55

the current I_y flows through the armature conductors connected between segments No. 1 and No. 2, joins I_x, and the combined current ($i_1 = I_x + I_y$) flows from a to b through the brush. When the brush is centered (T seconds later) on segment No. 2, the current I_x flows through the armature conductors connected between segments No. 1 and No. 2, joins I_y, and the combined current ($i_2 = I_x + I_y$) flows from c to b through the brush.

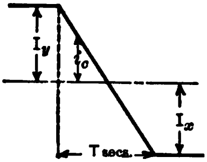


FIG. 56

It is then evident that the current (i_c) flowing in the armature conductors connected between segments No. 1 and No. 2 reverses during the period of commutation as shown in Fig. 56. Sparkless commutation will be obtained if the current (i_1) flowing from segment No. 1 into the brush is zero at the instant when the brush breaks contact with segment No. 1. Since the connection between a and b through segment No. 1 contains no source of e.m.f. the current (i_1) flowing in this connection will be zero when v_{ab} is zero. The potential between a and b (taking the path acb), from 64, is given by $v_{ab} = + e_c - i_c r_c - i_2 r_2 - L_c \frac{di_c}{dt}$, where e_c, i_c, r_c and L_c are the electromotive force, current, resistance and inductance respectively of the armature conductors connected between segments No. 1 and No. 2. Sparkless commutation will then be obtained if the e.m.f. (e_c) generated in the conductors undergoing commutation equals $i_c r_c + i_2 r_2 + L_c \frac{di_c}{dt}$ and acts from right to left in Fig. 55.

The brushes must therefore be located with reference to the poles in such a position that the armature conductors undergoing commutation will generate the required e.m.f. (e_c) as the brush breaks contact with segment No. 1. In a generator this implies that the brushes be advanced in the direction of rotation and in a motor retarded in the opposite direction as shown in Fig. 57. The angle between the commutating plane and the interpolar plane (θ in Fig. 57) must be increased as the load increases for two reasons: (1) because the distribution of the air-gap flux changes as the load increases and causes e_c to decrease, and (2) because i_c, i_2 and $\frac{di_c}{dt}$ increase with an increase of load and e_c must be increased in proportion to make v_{ab} equal zero as the brush breaks contact with segment No. 1.

It is often desirable that sparkless commutation be obtained without moving the brushes back and forth with each change of the load. Among the various methods proposed for obtaining sparkless commutation at all loads without advancing or retarding the brushes some are expected only to decrease the sparking while others may eliminate sparking completely. The higher the resistance of the brush contact for a given potential between *a* and *b* the lower the current will be flowing from *a* to *b* through segment No. 1 (Fig. 55) just before the brush breaks contact with segment No. 1. Carbon brushes present a higher contact resistance than copper brushes and are therefore employed on most dynamos to reduce the destructive action of sparking. The mechanical wear of a copper commutator is also less with carbon brushes than with copper brushes.

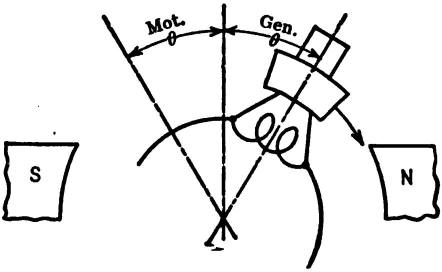


FIG. 57

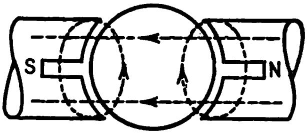


FIG. 58

Any measure which prevents a change in the distribution of the air-gap flux with a change in load will lessen the sparking evil since the e.m.f. generated in the armature conductors undergoing commutation will be the same at all loads if the field flux does not change with the load. In some dynamos the poles are slotted as shown in Fig. 58. The flux due to the armature magnetomotive force is then decreased by the high reluctance of the pole slots and the distortion of the air-gap flux is considerably reduced. The effect of the armature magnetomotive force may be completely eliminated by constructing a fixed winding about the armature, which is connected in series with the armature, and produces a magnetomotive force at all loads, equal and opposite to that of the armature winding. This fixed winding, shown in Fig. 59, is called a compensating winding.

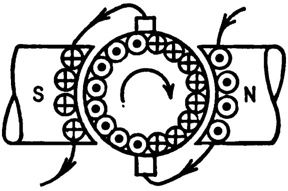


FIG. 59

Complete suppression of sparking at all loads may only be attained by making e_c equal to $i_c r_c + i_2 r_2 + L_c \frac{di_c}{dt}$ for any load. This result may be accomplished by constructing auxiliary poles, called commutating poles, between the main poles as shown in Fig. 60. The commutating pole windings are connected in series with the armature so that the commutating pole flux will increase with the load. The commutating pole flux may then be adjusted so that with the brushes located in a fixed position an e.m.f. will be generated in the armature conductors undergoing commutation, as they pass through the commutating pole flux, which will make v_{ab} (Fig. 55) equal zero at the proper instant and produce sparkless commutation at all loads. The fixed adjustment of the commutating pole flux is usually accomplished by means of a variable resistance connected in parallel with the commutating pole winding.

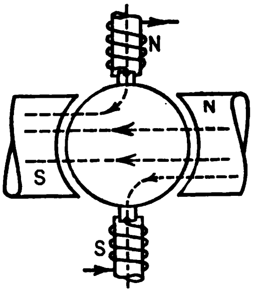


FIG. 60

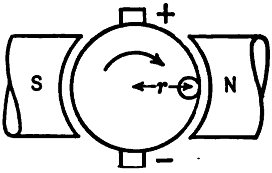


FIG. 61

Difference between a Generator and a Motor. The direction of the e.m.f. generated in an armature conductor at the instant shown in Fig. 61, by the right-hand rule, is downward. If the direction of the current flowing in this conductor is also downward, the direction of the force acting on the conductor, by the left-hand rule, is counterclockwise or opposite to the stated direction of rotation. The rotation of the armature must therefore be due to some external torque. Mechanical energy is being converted into electric energy and the dynamo is operating as generator. If the direction of the current flowing in the conductor is upward the direction of the force acting on the conductor, by the left-hand rule, is clockwise or in the same direction as the stated direction of rotation. The rotation of the armature is therefore due to the electromagnetic force acting on the conductor. Electric energy is being converted into mechanical energy and the dynamo is operating as a motor. The nature of the operation of a dynamo is thus dependent solely upon the relative directions of the current and e.m.f. in its armature conductors; current and e.m.f. in the same direction indicate generator action, and current and e.m.f. in the opposite direction motor action.

The relative direction of the current and generated e.m.f. in the armature conductors of a dynamo depends upon the relative magnitude of the terminal potential and the generated e.m.f. When the generated e.m.f. (E_a) of a dynamo is greater than the terminal potential (V_t) the armature current, from 14, is given by $I_a = \frac{E_a - V_t}{R_a}$. The current then flows in the same direction as the generated e.m.f. in each armature conductor and the dynamo operates as a generator. The terminal potential of a generator, from 6, is then given by

71 $$V_t = E_a - I_a R_a \text{ volts.}$$

When the generated e.m.f. (E_a) of a dynamo is less than the terminal potential (V_t), the armature current, from 14, is given by $I_a = \frac{V_t - E_a}{R_a}$. The current then flows in the opposite direction to the generated e.m.f. in each armature conductor and the dynamo operates as a motor. The terminal potential of a motor, from 6, is then given by

72 $$V_t = E_a + I_a R_a \text{ volts.}$$

Electromagnetic Torque of a Dynamo Armature. If the average flux density normal to the surface of an armature is $B_{av.}$ gausses, the armature current is I_a amperes, the flux-cutting length of each armature conductor is s centimeters and there are m paths in the armature between terminals, the average force acting upon each armature conductor, from 36, is given by $F_{av.} = \frac{B_{av.} I_a s}{10\, m}$ dynes. If the distance from each conductor to the center of the armature (Fig. 61) is r centimeters, the average turning moment or torque produced by each conductor is given by $T_{av.} = \frac{B_{av.}I_a sr}{10\,m}$ centimeter-dynes and the total torque due to Z conductors is given by $T = \frac{ZB_{av.}I_a sr}{10\,m}$ centimeter-dynes. Since the product of the number of poles (p) and the flux entering or leaving each pole (Φ) equals the surface area of the armature permeated by magnetic flux ($2\,\pi rs$) multiplied by the average flux density on the surface ($B_{av.}$), $p\Phi = 2\,\pi rs\,B_{av.}$ and $B_{av.}sr = \frac{p\Phi}{2\,\pi}$. Substituting $\frac{p\Phi}{2\,\pi}$ for $B_{av.}sr$ and converting the torque units into pound-feet, the electromagnetic torque of a dynamo is given by

73 $$T = \frac{1.175\,ZI_a p\Phi}{m \times 10^9} \text{ pound-feet.}$$

In a generator the driving torque must exceed the value given by 73 and in a motor the torque developed at the pulley will be less than the value given by 73, the difference in each case being the torque associated with the rotational losses of a dynamo (discussed in detail on page 62).

The Speed-Torque Characteristic of a Motor. While the speed of a generator armature is fixed by the speed of the prime mover that drives it, the speed of a motor armature depends upon the electric and magnetic conditions within the armature and the nature of the mechanical load. The electromagnetic torque developed by a motor, from 73, may be indicated by $T = K_1\Phi I_a$ pound-feet, where $K_1 = \frac{1.175\,Zp}{m \times 10^9}$. The armature current, from 14, is given by $I_a = \frac{V_t - E_a}{R_a}$ and, since $E_a = K_2\Phi S$ where $K_2 = \frac{Zp}{6\,m \times 10^9}$, from 68, $I_a = \frac{V_t - K_2\Phi S}{R_a}$. Hence the electromagnetic torque developed by a motor when the armature speed is S revolutions per minute is given by

74 $$T = \frac{K_1\Phi}{R_a}\,(V_t - K_2\Phi S) \text{ pound-feet.}$$

The electromagnetic torque is evidently a maximum at standstill unless the flux for some reason decreases with the speed. The actual standstill torque will be less than that indicated by 74 because resistance (R) must be connected in series with the armature at standstill so that $I_a = \frac{V_t}{R_a + R}$ will not greatly exceed the armature current rating of the motor.

The running speed of a motor is that speed, substituted in 74, which will make the electromagnetic torque equal the reactive torque, or the sum of the load torque and the rotational loss torque. A motor thus eliminates automatically any difference between the electromagnetic torque and the reactive torque by a suitable change in speed. Since the flux may also change with the speed the magnitude of the change in speed in any readjustment may depend upon the simultaneous change of flux. Since $K_2\Phi S = V_t - I_aR_a$ and R_a is usually small $K_2\Phi S$ is sensibly constant, decreasing slightly as I_a increases. The change of speed due to a change of flux is therefore greater than a change of speed due to a change in reactive torque. A change of reactive torque alone will produce a slight change in speed but a change of flux in the ordinary motor will produce an inverse change in speed of nearly equal proportion.

The Shunt Motor. The shunt field current of a shunt motor is supplied from the line as shown in Fig. 62. Since the terminal potential (V_t) and the shunt field resistance (R_f) are in general constant the shunt field current (I_f), given by $I_f = \frac{V_t}{R}$, will also be constant. The air-gap flux of a shunt motor will then be sensibly constant, changing slightly with the armature reaction. The flux (Φ) in 74 being assumed constant the torque of a shunt motor may be indicated by $T = K - K'S$ and may be represented graphically, within the range of current rating, by the curve shown in

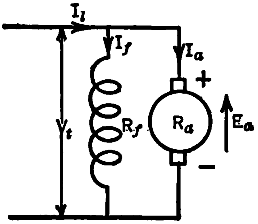


FIG. 62

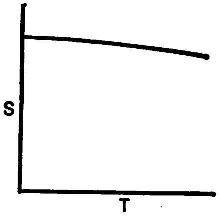


FIG. 63

Fig. 63. Since the speed decreases slightly as the torque increases the shunt motor is employed in most systems of electric drive which require variable torque at sensibly constant speed.

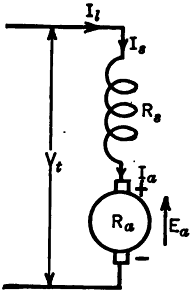


Fig. 64

The Series Motor. In the series motor (Fig. 64) the line current (I_l), the series field current (I_s) (if the series field is not shunted) and the armature current (I_a) are the same. The air-gap flux will then vary with the line current (I_l) and, since $E_a = K_2\Phi S$ is a constant (very nearly) $\Phi S = K$ or $\Phi = \frac{K}{S}$ approximately. Substituting $\frac{K}{S}$ for Φ in 74, $T = \frac{K'}{S}$ (approx.). The torque of a series motor is then (approx.) inversely proportional to the speed as shown in Fig. 65. If the no load losses are small the no load torque will be small and the no load speed may produce excessive centrifugal stresses in the armature. The operation of series motors of more than one horse-power capacity at no load is therefore inadvisable. Since the torque increases as the speed decreases as shown in Fig. 65 the series motor is best adapted to applications of the electric drive which require light torque at high speed and heavy torque at low speed, and which never allow the motor to operate at no load. Street cars are invariably driven by series motors for this reason.

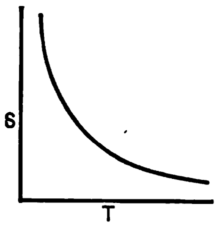


Fig. 65

The Compound Motor. Motors having both shunt and series field windings, called compound motors, possess speed-torque characteristics which may prove more desirable than those of the shunt or series motors in certain applications. Two types of compound motor may be used: the cumulative compound and the differential compound.

In the cumulative compound motor (Fig. 66) the shunt and series magnetomotive forces act in conjunction. The relationship of the flux and speed may then be expressed by $\Phi = K + \frac{K'}{S}$ (approx.) since the flux due to the shunt field is constant and the flux due to the series field varies (approx.) inversely as the speed. Substituting $K + \frac{K'}{S}$ for Φ in 74, $T = K_1 + \frac{K_2}{S} - K_3S$. This relationship may be represented typically by the curve shown in Fig. 67. The slope of the curve depends upon the relative magnitude of the shunt and series magnetomotive forces. It will be noted that the cumulative compound motor has a definite (safe) no load speed and resembles the shunt motor in this respect but operates at lower speed than the shunt motor at higher values of the torque. The cumulative compound motor is therefore employed in electric drives which require a definite no load speed and a lower speed than would be furnished by the shunt motor under heavy torque.

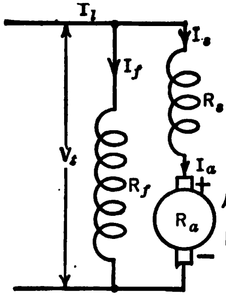


FIG. 66

In the differential compound motor shown in Fig. 68 the shunt and series field magnetomotive forces act in opposition. The relationship of the air-gap flux and the speed may then be expressed by $\Phi = K - \frac{K'}{S}$ (approx.), since the flux due to the shunt field is constant and the flux due to the series field (in opposition) varies (approx.) inversely as the speed.

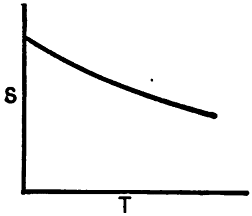


FIG. 67

Substituting $K - \frac{K'}{S}$ for Φ in 74, $T = K_1 - \frac{K_2}{S} - K_3S$. This relationship may be represented typically by the curve shown in Fig. 69. The no load speed is definite as in the shunt and cumulative compound motor. An increase of load strengthens the series field flux (which opposes the constant shunt field flux) and decreases the air-gap flux. The generated e.m.f. ($K_2\Phi S$) being sensibly constant the speed must increase. Since an increased speed will in general increase the reactive torque of the load the speed will increase consecutively resulting in unstable operation. The starting torque is also low (it may be negative and start the motor in the wrong direction) since the opposing series field under starting conditions may produce a flux equal to or greater than that due to the shunt field. A differential compound motor with very few series turns may operate at sensibly constant speed but a shunt motor will serve nearly as well for this purpose and possesses none of the disadvantages of the differential compound motor.

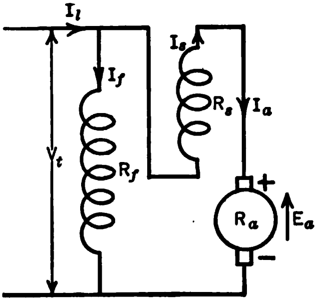


FIG. 68

The Control of Starting Current in a Motor. The current flowing in the armature of a motor connected directly to the line at standstill is given, from 14, by $I_a = \frac{V_t}{R_a}$ for a shunt motor and by $I_a = \frac{V_t}{R_a + R_s}$ for a series or a long-shunt compound motor. In most motors this current would be at least ten times greater than the rated armature current of the motor at full load. In a motor thus connected directly to the line the armature winding and the brush contacts would be heated excessively, the armature winding would be subjected to excessive mechanical stresses, and the heavy current drawn from the source of power supply would blow fuses or open circuit breakers, change the terminal potential and speed of other motors and dim the electric lamps on the same circuit.

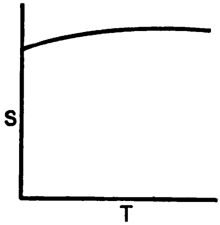


FIG. 69

Before a motor is connected to the line a resistance (R) should therefore be connected in series with the armature so that the initial current given by $I_a = \frac{V_t}{R_a + R}$ or $\frac{V_t}{R_a + R_s + R}$ will not greatly exceed the rated armature of the motor at full load. The various types of starting box employed for this purpose contain some arrangement for decreasing the resistance connected in series with the armature as the motor comes up to speed and in general provide for an automatic reconnection of the series resistance when the line switch is opened so that the armature will be protected when the switch is closed again.

A common type of shunt motor starting box is shown in Fig. 70.

After closing the line switch (*S*), the starting handle (*H*) is moved toward the right. The resistance (*XY*) in the starting box connected in series with the armature is thus gradually reduced to zero as the motor comes up to speed. Current also flows through the starting handle to *X* and then through a small electromagnet (*C*) on the face of the starting box to the shunt field. The resistance removed from the armature circuit is thus transferred to the field circuit during the starting period so that the initial field current may exceed the normal field current and provide increased starting torque. When the starting handle makes contact with *Y* (the motor is then running at rated speed with all starting resistance disconnected) the handle is held in that position by the magnetic attraction of the electromagnet (*C*). When the line switch is opened the current in the electromagnet (*C*) is interrupted and a helical spring at *O* returns the starting handle to the original starting position.

A common type of series motor starting box is shown in Fig. 71. The construction and operation is the same as in the shunt motor starting box except that the electromagnet (*C*) of high resistance wire is connected across the line (from *L* + to *L* −). The opening of the line switch interrupts the current in the electromagnet (*C*) and the helical spring at *O* returns the starting handle to the original starting position.

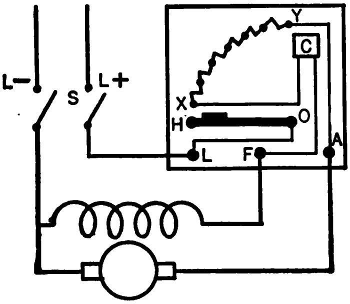


FIG. 70

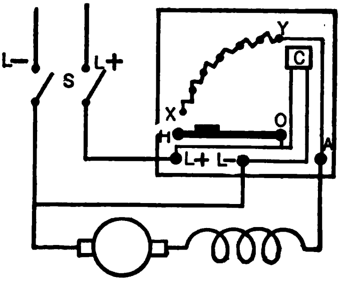


FIG. 71

Adjustment of the Speed of a Motor. Since the speed of any motor is maintained by its electromagnetic torque a momentary change in the electromagnetic torque is followed by a similar change in speed. The electromagnetic torque, from 74, being given by $T = \frac{K_1\Phi}{R_a}(V_t - K_2\Phi S)$ the torque and hence the speed of any motor must depend (other factors remaining constant) directly upon V_t and inversely upon R_a. The speed of any motor may then be increased by increasing V_t or by decreasing R_a if the flux (Φ) is maintained constant. It will be noted that a change in Φ may produce either a momentary increase or decrease in T depending upon the relative magnitude of V_t and $K_2\Phi S$.

When $K_2\Phi S$ (or E_a) is greater than $\frac{V_t}{2}$ (as it is in the ordinary motor) an increase of Φ is followed by a momentary decrease of T and a consequent decrease in S. If $K_2\Phi S$ (or E_a) is less than $\frac{V_t}{2}$ an increase of Φ is followed by a momentary increase of T and a consequent increase in S. In the ordinary motor, since E_a nearly equals V_t, the speed is therefore inversely proportional to the flux. The speed of a shunt motor may be increased by increasing the resistance of the shunt field circuit and the speed of a series motor may be increased by decreasing the resistance of a shunt connected in parallel with the series field. If the armature resistance in either motor is of such magnitude that E_a is less than $\frac{V_t}{2}$ the speed changes, for the stated changes of resistance will be reversed.

The methods of adjusting the speed of a motor may be summarized as follows:

Increase	Increase	Decrease
Terminal potential	Speed	....
Armature resistance	....	Speed
Air-gap flux $\left(E_a > \frac{V_t}{2}\right)$	....	Speed
Air-gap flux $\left(E_a < \frac{V_t}{2}\right)$	Speed	....

If two or more factors are changed simultaneously the resulting change in speed will depend upon the magnitude of each effect. In the shunt motor, for example, an increase of V_t will also cause an increase of Φ. If $E_a > \frac{V_t}{2}$ the two effects will be in opposition and the amount of speed increase will depend upon the flux density in the magnetic circuit, the increase of speed for a given increase of V_t being greater when the magnetic circuit is saturated.

The speed change due to any number of changes in the operating conditions of a motor may be determined by equating ratios of E_a in 72 and 68 under the two operating conditions as follows:

75 $$\frac{(V_t - I_aR_a)_1}{(V_t - I_aR_a)_2} = \frac{\left(\frac{P\Phi ZS}{m}\right)_1}{\left(\frac{(P\Phi ZS)}{m}\right)_2}.$$

The Power Losses in a Dynamo. The power losses in a dynamo are usually classified as (1) copper losses and (2) rotational losses (frequently called the stray power). The copper losses (the power converted into heat in the shunt field, series field and armature windings) are determined as follows:

76 Shunt field $P_f = V_fI_f = I_f^2R_f = \frac{V_f^2}{R_f}$ watts,

77 Series field $P_s = I_s^2R_s$ watts,

78 Armature $P_a = I_a^2R_a$ watts.

The rotational losses include (1) friction losses in the bearings and at the brushes, (2) windage loss due to the motion imparted to the surrounding air by the rotating armature, (3) hysteresis loss (see page 24) due to the reversing magnetism in the rotating armature core and (4) eddy current loss due to the heating effect of currents circulating in the armature core.

Reduction of the Losses in a Dynamo. The copper losses depend upon the square of the current and the resistance of the various windings. Since the ampere turns required in the field windings depend upon the reluctance of the magnetic circuit the field copper losses will depend upon the cross-sectional area of the magnetic circuit. The reduction of the copper losses (since the currents in the various windings of a given dynamo are sensibly fixed) may be accomplished principally by increasing the weight of copper and iron in the respective electric and magnetic circuits. Copper loss reduction is therefore limited by the desired weight and cost of the dynamo and by the desired magnitude of the rotational losses which increase in general as the weight of the dynamo increases.

With reference to the rotational losses, the friction loss depends principally upon the speed, the condition of the bearings and the commutator, the lubrication and the brush pressure. The windage loss depends upon the speed, the surface irregularities of the armature and the nature of the armature enclosure. Since the circulation of air around the armature assists in cooling the various windings and the armature core no attempt is made to reduce the windage loss. Fan blades are frequently attached to the armature to augment the circulation of air and increase the rated capacity of the dynamo.

The hysteresis loss may be computed with certain assumptions by

79 $$P_h = \frac{\eta p S V B_m^{1.6}}{12 \times 10^8} \text{ watts,}$$

where $\eta = 0.004$ for ordinary sheet iron and 0.001 for the best annealed sheet iron, p = the number of poles, S = the speed in r.p.m., V = the volume of the armature core in cubic centimeters, and B_m = the average maximum flux density in the armature core in gausses. The eddy current loss may also be computed with certain assumptions by

80 $$P_e = \frac{V\ (tpSB_m)^2}{87.8\ \rho \times 10^8} \text{ watts,}$$

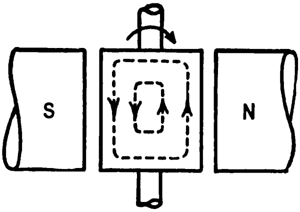


Fig. 72

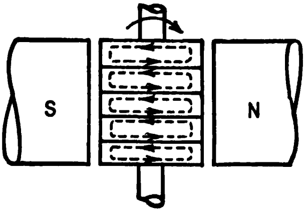


Fig. 73

where t = the thickness of the laminations in centimeters and ρ = the resistivity of the iron in ohms per centimeter cube. All other quantities are described under hysteresis loss. If the armature core were solid the e.m.f.'s generated in the rotating core would cause currents to flow in the iron as shown in Fig. 72. By constructing the core of iron sheets (called laminations) insulated from each other electrically as shown in Fig. 73 the ratio of the e.m.f. to the resistance in each lamination is reduced and the eddy current loss is materially diminished. The laminations are punched from soft sheet iron, 0.015 inch or more in thickness, and are insulated by the natural surface oxide or by a coat of varnish.

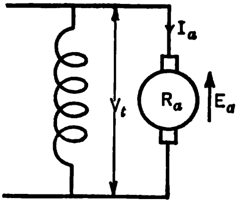


FIG. 74

Determination of the Rotational Losses in a Dynamo. The rotational losses are usually determined experimentally by operating the dynamo as a shunt motor at no load, as in Fig. 74. The input to the armature (V_tI_a) minus the armature copper loss ($I_a^2R_a$) minus the rotational losses (P_r) equals zero or $V_tI_a - I_a^2R_a - P_r = 0$ and

81 $$P_r = V_tI_a - I_a^2R_a \text{ watts.}$$

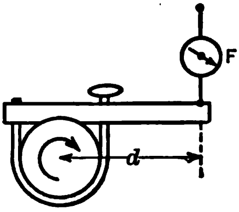


FIG. 75

Since the rotational losses depend upon the speed and the flux density the rotational losses corresponding to any stated load are determined by operating the motor at no load at the same speed and flux density as at the stated load. If the mechanical output (P_o) of a motor operated under load may be measured accurately the rotational losses may be determined by

82 $$P_r = V_tI_a - I_a^2R_a - P_o \text{ watts.}$$

The Efficiency of a Dynamo. The efficiency of a generator or a motor equals the ratio of the power output to the power input. The efficiency of a generator is then given by the ratio of the power output to the sum of the power output and the losses. Hence the efficiency of a generator is given by

83 $$\eta_g = \frac{V_tI_l}{V_tI_l + V_fI_f + I_s^2R_s + I_a^2R_a + P_r}.$$

The efficiency of a motor is given by the ratio of the power input minus the losses to the power input or

84 $$\eta_m = \frac{V_tI_l - V_fI_f - I_s^2R_s - I_a^2R_a - P_r}{V_tI_l}.$$

If the output of a motor is measured by a Prony brake, as in Fig. 75, then $P_o = \frac{2\pi\, dSF}{33{,}000}$ horse power, measuring d in feet, F in pounds and S in revolutions per minute. Substituting T, the torque in pound-feet, for dF the power output is given by

85 $$P_o = 1.903\, ST \times 10^{-4} \text{ horse power},$$

86 $$P_o = 0.1420\, ST \text{ watts}.$$

The efficiency of a motor is then given by

87 $$\eta_m = \frac{0.1420\, ST}{V_t I_l}.$$

Characteristic Curves of a Motor. The principal operating characteristics of a motor (speed, efficiency, torque and line current) are usually plotted with reference to the output in horse power and are called the characteristic curves of a motor. Typical characteristic curves of a shunt motor are shown in Fig. 76, and of a series motor in Fig. 77. Full load values for each quantity are indicated by the intersection of the vertical dotted line.

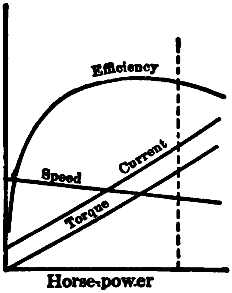


FIG. 76

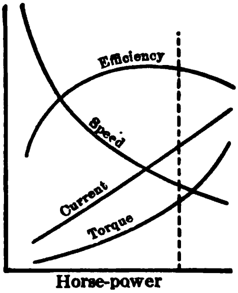


FIG. 77

CHAPTER V

DIRECT CURRENT MEASUREMENTS

Legal Units of Electrical Measure. An Act of Congress (1894) following the recommendation of the International Electrical Congress of 1893 established the following legal units of resistance, current and electromotive force in the United States.

The Ohm. The resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14.4521 grams in mass, of constant cross-sectional area and of the length 106.3 centimeters.

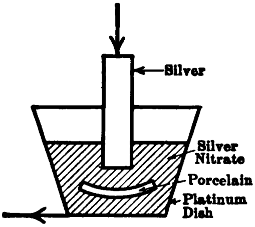


FIG. 78

The Ampere. The unvarying current which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of 0.001118 gram per second.

The Volt. The electromotive force that, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere.

The same units have been adopted by most other countries and are therefore called international units. The device in which current is measured by the deposition of silver is called a silver voltameter (Fig. 78).

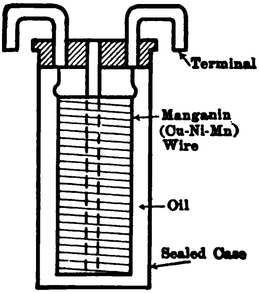


FIG. 79

Secondary Reference Standards. Since the mercury ohm and the silver voltameter have proved inconvenient for frequent use as reference standards the Bureau of Standards has prepared and maintained secondary reference standards of resistance and electromotive force. The secondary standards of resistance are sealed wire resistances called N. B. S. (National Bureau of Standards) standard resistances

(Fig. 79), which are compared at infrequent intervals with the mercury ohm. The secondary standard of electromotive force is the saturated Weston cell (Fig. 80) which is calibrated at infrequent intervals by the silver voltameter and a standard resistance. The e.m.f. of this cell at 20 degs. Cent. is 1.0183 volts. When used as reference standards the secondary standards of resistance and e.m.f. are compared respectively in groups, the average value of the group in each case being adopted as the secondary standard.

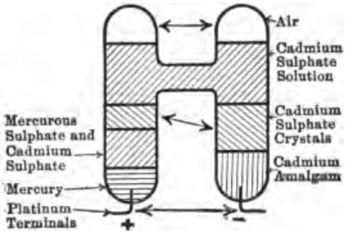

Air
Cadmium Sulphate Solution
Cadmium Sulphate Crystals
Cadmium Amalgam
Mercurous Sulphate and Cadmium Sulphate
Mercury
Platinum Terminals
+
−

FIG. 80

Commercial Laboratory Standards. The reference standards of the commercial laboratory resemble the secondary standards of the Bureau of Standards but may differ in certain elements of construction. A common form of commercial standard resistance, called the Reichsanstalt model, is a wire standard but unlike the N. B. S. model is not hermetically sealed. The commercial standard cell is usually the unsaturated Weston cell, the cell used by the Bureau of Standards being saturated; that is, the Bureau of Standards cell contains an excess of cadmium sulphate crystals. While not as reliable as the saturated cell, the unsaturated cell is less affected by changes of temperature. The commercial or tertiary reference standards are compared occasionally with groups of secondary reference standards at the Bureau of Standards so that any changes in them may be detected.

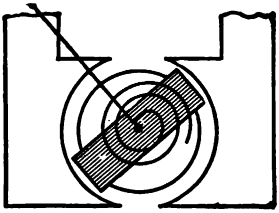


FIG. 81

Ammeters and Voltmeters. Portable or switchboard instruments designed for the measurement of current or potential usually consist of a light rectangular coil of fine copper wire wound upon an aluminum frame, pivoted in jeweled bearings and located in the annular space between a soft iron core and the pole faces of a permanent magnet as shown in Fig. 81. Electrical connections with the coil are made through two spiral springs which also resist the angular motion of the coil with a torque proportional to the angular twist. Since the torque acting upon the coil is proportional to the coil current the magnitude of the current is indicated by the motion of a light tubular pointer over a calibrated scale.

The instrument shown in Fig. 81 may be employed to measure a current not greater than 0.025 ampere and a potential not greater than 1.5 volts (approx.), the potential being determined indirectly by the product of the coil current and the coil resistance. To measure a current greater than 0.025 ampere the coil is shunted as shown in Fig. 82. If the coil resistance is represented by R_c and the shunt resistance by R_s the ammeter resistance is given by $R_a = \frac{R_cR_s}{R_c + R_s}$. The ammeter current is then given by $I_a = \frac{I_cR_c}{R_a}$, the range of the instrument being dependent upon the relative magnitude of R_c and R_a. To measure a potential greater than 1.5 volts the coil is connected in series with a high resistance as shown in Fig. 83. If the resistance of the coil is represented by R_c and the added series resistance by R_e the resistance of the voltmeter is given by $R_v = R_e + R_c$. The potential (V_v) across the voltmeter in terms of the potential (V_c) across the coil is then given by $V_v = \frac{V_cR_v}{R_c}$, the range of the instrument being dependent upon the relative magnitude of R_v and R_c.

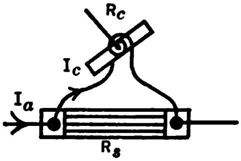


FIG. 82

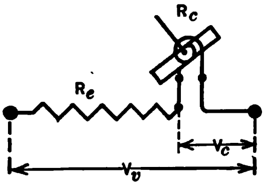


FIG. 83

The Wattmeter. The measurement of power in a direct current circuit is seldom accomplished by a single instrument since the power may always be determined by the product of the voltmeter and ammeter readings. The dynamometer wattmeter described on page 92 may be employed for this purpose whenever the use of a single instrument is found desirable.

The Watt-hour Meter. The instrument most commonly employed for measuring the energy supplied to any part of a direct current circuit is the Thomson watt-hour meter shown in Fig. 84. The revolving element consists of a spherical armature (*A*) with a commutator (*C*) and a metal disc (*D*), all mounted upon a shaft which turns on a jeweled bearing (*B*). The number of revolutions of the revolving element is registered by a series of dials (*G*) which are connected by gears to the shaft. Two field coils (*FF*) are connected in series with the load under measurement, and the armature in series with a compensating coil (*C'*) and a high resistance (*H*) is connected across the line. The metal disc (*D*) revolves between the pole faces of two or more permanent magnets (*MM*).

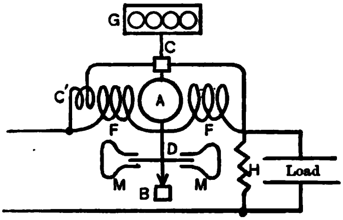


FIG. 84

The instrument consists essentially of a direct current shunt motor (at the top) which drives an alternating current generator (at the bottom). In the motor the field flux (Φ_f) is proportional to the line current (I_l) and the armature current (I_a) is proportional to the line potential (V_l) since the e.m.f. generated in the armature is very small. The motor torque (T_M) is then proportional to V_lI_l or the power (P) supplied to the load, and $T_M = K_1P$. In the generator, since the magnetic flux passing through the rotating disc is constant, the e.m.f. (E) generated in the disc is proportional to the speed (S) of the disc. Since the resistance of the disc is constant for a given temperature the current (I) flowing in the disc is also proportional to the speed (S). The flux passing through the disc being constant the generator torque (T_G) is then proportional to the speed (S), and $T_G = K_2S$.

If the friction of the bearing, commutator and gears is neglected $T_M = T_G$ and $P = KS$. Since energy (W) equals the product of power (P) and time (t), $W = Pt = KSt$ and $W = KR$ where $R = St$ is the number of revolutions of the disc in time (t). With the proper gear connection the dials will then indicate directly the energy supplied to the load in a given time. The frictional torque of the revolving element being too large to be neglected, a compensating field coil (C') is provided to supply an additional torque which is independent of the load and equal to the frictional torque. Owing to the high resistance of the armature circuit the speed of the motor is directly proportional to the field flux as described on page 61.

Measurement of Resistance. Accurate measurement of resistance may be accomplished by some form of Wheatstone bridge as shown in Fig. 85. A battery (*B*) is connected to the terminals (*ab*) of a parallel connection containing three known adjustable resistances (R_1, R_2 and R_3), the unknown resistance (R_4), and a galvanometer (*G*) connected between *c* and *d*.

The galvanometer usually consists of a light coil of fine wire suspended between the poles of a permanent magnet as shown in Fig. 86. When conditions demand a more sensitive instrument a small permanent magnet is suspended at the center of a coil of wire as shown in Fig. 87. In either case the moving element of the galvanometer will twist when a very small current is sent through the galvanometer coil. Evidence of a twist in the moving element is usually magnified by directing a beam of light upon a small mirror attached to the moving element so that the reflected light may be viewed on a scale located at some distance from the galvanometer. The galvanometer in this way serves as a sensitive detector of an electric current.

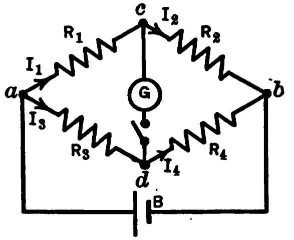


FIG. 85

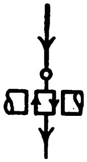


FIG. 86

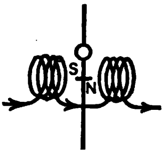


FIG. 87

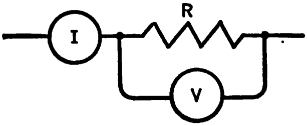


FIG. 88

If no galvanometer deflection follows the closing of the galvanometer key in Fig. 85 the potential between *c* and *d* must be zero. Then $I_1 = I_2$, $I_3 = I_4$, $I_1R_1 = I_3R_3$, $I_2R_2 = I_4R_4$, $\frac{I_1R_1}{I_2R_2} = \frac{I_3R_3}{I_4R_4}$ and $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. The value of R_4 may thus be determined by adjusting R_1, R_2 and R_3 until there is no deflection of the galvanometer.

A less accurate measurement of resistance may be made with a voltmeter and an ammeter connected as shown in Fig. 88. If the current taken by the voltmeter is neglected $R = \frac{V}{I}$ ohms. If the current taken by the voltmeter of R_v ohms resistance is considered $R = \frac{VR_v}{IR_v - V}$ ohms.

The Potentiometer. Accurate laboratory measurements of potential and current, the calibration of instruments, etc., are in general accomplished by means of the instrument shown in Fig. 89

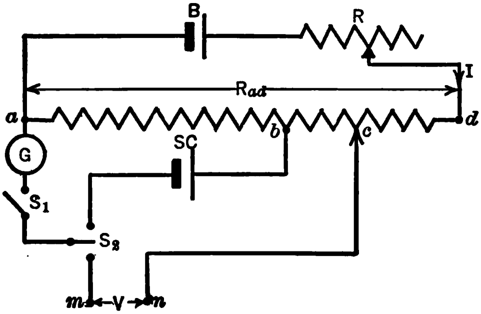


FIG. 89

called a potentiometer. A battery (B) sends a current through an adjustable resistance (R) and a fixed resistance (R_{ad}), the capacity of the battery being sufficient to maintain a steady current in the fixed resistance. One terminal of a standard cell (SC) is connected permanently to a definite point (b) in the fixed resistance (R_{ad}) and the other terminal may be connected through a double-throw switch (S_2), a galvanometer key (S_1) and a galvanometer (G) to the point (a). If the resistance (R) is adjusted until the galvanometer shows no deflection with the standard cell connected in the circuit IR_{ab} must equal E_{sc}, the e.m.f. of the standard cell; that is, V_{ab} equals 1.0183 volts (usually).

If the double-throw switch is now thrown downward connecting one terminal of an unknown potential (V) through the galvanometer to (a) and the other terminal of the unknown potential (V) is moved along the fixed resistance (R_{ad}) until the galvanometer shows no deflection, then V must equal V_{ac} or $V = 1.0183 \left(\frac{R_{ac}}{R_{ab}}\right)$.

The ratio $\left(\frac{R_{ad}}{R_{ab}}\right)$ is usually such as to make V_{ad} equal 1.5 volts when the connection of the standard cell in circuit produces no deflection of the galvanometer. The resistance (R_{ad}) is divided into several thousand equal parts and the end of each division is marked with the value $1.0183\left(\frac{R_{ax}}{R_{ab}}\right)$, R_{ax} being the resistance from (a) to any point (x). The value of the unknown potential may then be read directly from the instrument at the point where the contact (c) gives no deflection of the galvanometer.

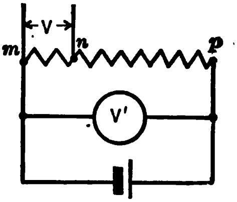


FIG. 90

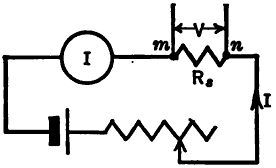


FIG. 91

Potentials higher than 1.5 volts may be measured by connecting a definite part of a high resistance (mp) between (m) and (n) as shown in Fig. 90. The unknown potential (V') then equals $V\left(\frac{R_{mp}}{R_{mn}}\right)$ and a voltmeter connected between (m) and (p) may be calibrated throughout its range. In measuring current with the potentiometer a standard resistance (R_s) is connected between (m) and (n) as shown in Fig. 91. The current flowing through the standard resistance is then given by $I = \frac{V}{R_s}$ and an ammeter connected in series with the standard resistance may be calibrated throughout its range.

CHAPTER VI

ALTERNATING CURRENT CIRCUITS

Alternating Electromotive Force. The variation of the instantaneous e.m.f. generated in each armature conductor of a dynamo during one revolution of the armature depends upon the distribution of the flux density in the air gap as explained on page 37. A plot of the successive instantaneous values of the e.m.f. generated during the passage of a conductor by two poles is called the wave form of the e.m.f., a typical wave form of an alternating e.m.f. being shown in Fig. 92. It is desirable for many reasons that the wave form be sinusoidal and it will be so assumed throughout the chapter. A source of alternating e.m.f. is indicated by the symbol ∿.

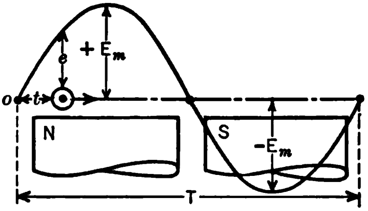


FIG. 92

Terminology of an Alternating E.m.f. A complete sequence of values of e.m.f. from zero to maximum positive ($+E_m$) to zero to maximum negative ($-E_m$) to zero (Fig. 92) is called the e.m.f. cycle. The time in seconds taken by the e.m.f. to pass through one cycle is called the period (T) of the e.m.f. cycle. The number of e.m.f. cycles per second is called the frequency (f) of the e.m.f. cycles. The frequency then equals the reciprocal of the period or $f = \frac{1}{T}$ cycles per second. The frequency of the e.m.f. and current in most alternating current systems is either 60 or 25 cycles per second, 60 cycles per second being employed generally in lighting circuits and 25 cycles per second in circuits supplying power to motors.

The mathematical equation for a sine wave is $y = a \sin bx$. If the angle subtended by two poles is called 2π radians and the origin be located at 0 (Fig. 92), the angular distance of the conductor from the origin t seconds after leaving the origin will be $\frac{t}{T} \cdot 2\pi$ radians.

Substituting e for y, E_m for a and $\frac{2\pi}{T} \cdot t$ for bx, the equation of the e.m.f. is given by $e = E_m \sin \frac{2\pi}{T} t$ or $e \doteq E_m \sin 2\pi f t$. The product $(2\pi f)$ which represents the angular velocity of the conductor is usually indicated by ω. The mathematical equation for a sinusoidal e.m.f. is then given by

88 $$e = E_m \sin \omega t \text{ volts,}$$

where e is the e.m.f. in volts generated in the conductor t seconds after the e.m.f. is zero and is increasing toward the positive maximum, E_m is the maximum value of the e.m.f. in volts, and ω is the angular velocity of the conductor in electrical radians per second. Electrical radians equal trigonometric radians multiplied by the number of pairs of poles.

Difference in Phase. The wave forms of the two e.m.f.'s generated in the two conductors shown in Fig. 93 will be identical since each conductor must pass through the same magnetic field. The e.m.f. generated in conductor No. 1 must differ from that generated in conductor No. 2 at the same instant since the two conductors do not pass through the same part of the magnetic field at the same instant. Corresponding values of e.m.f. are generated later in conductor No. 2 than in conductor No. 1 and the two e.m.f.'s are said to differ in phase by α degrees. The phase difference between e_1 and e_2 may be represented graphically as shown in Fig. 94 in which the two wave forms are displaced by the angle (α) between the two conductors. In the wave diagram a lagging e.m.f. by convention is displaced to the right and a leading e.m.f. to the left of the arbitrary axis of reference. Although the angle (ωt) in 88 is measured in radians a phase difference is usually expressed in degrees. The e.m.f. (e_2) in Fig. 94 for example, is

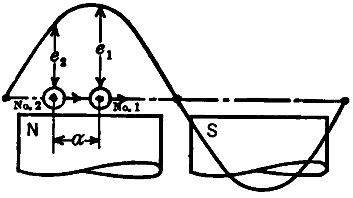


FIG. 93

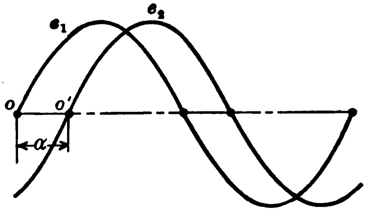


FIG. 94

said to lag the e.m.f. (e_1) by α degrees or the e.m.f. (e_1) is said to lead the e.m.f. (e_2) by α degrees. Phase difference may be indicated mathematically by referring each e.m.f. to the same origin. With the origin at o, $e_1 = E_{m1} \sin \omega t$ and $e_2 = E_{m2} \sin (\omega t - \alpha)$ and with the origin at o', $e_2 = E_{m2} \sin \omega t$ and $e_1 = E_{m1} \sin (\omega t + \alpha)$.

Addition of Sinusoidal E.m.f.'s. The two e.m.f.'s (e_1 and e_2) in Fig. 94 may be added graphically by adding simultaneous ordinates as shown in Fig. 95. The resultant e.m.f. (e_0) will also be sinusoidal and will differ in phase from both e_1 and e_2. The same e.m.f.'s expressed mathematically by

$$e_1 = E_{m1} \sin \omega t \quad \text{and} \quad e_2 = E_{m2} \sin (\omega t - \alpha)$$

may be added by applying the trigonometric formula for the sum of the sines of two angles multiplied by constants. The resultant e.m.f. is then given by

89 $$e_0 = \sqrt{(E_{m1} + E_{m2} \cos \alpha)^2 + (E_{m2} \sin \alpha)^2} \sin\left(\omega t - \tan^{-1} \frac{E_{m2} \sin \alpha}{E_{m1} + E_{m2} \cos \alpha}\right) \text{volts.}$$

The same result may be obtained in a simpler manner by representing each e.m.f. by a rotating vector as shown in Fig. 96, the rotation by convention being counter-clockwise at an angular velocity of ω radians per second. E_{m1}, the maximum value of e_1, is drawn along the horizontal axis (the axis of reference) and E_{m2}, the maximum value of e_2, is drawn at an angle of α degrees in a clockwise direction from E_{m1} to indicate a phase lag of α degrees. The instantaneous e.m.f. in either case is given by the projection of each vector on the vertical axis. Since each vector rotates at the same speed the maximum value of their sum in accordance with the principle of vector addition is given by the diagonal (E_{m0}) of the completed parallelogram. The horizontal component of E_{m0} is $E_{m1} + E_{m2} \cos \alpha$ and the vertical component is $E_{m2} \sin \alpha$. Then $E_{m0} = \sqrt{(E_{m1} + E_{m2} \cos \alpha)^2 + (E_{m2} \sin \alpha)^2}$. The phase angle

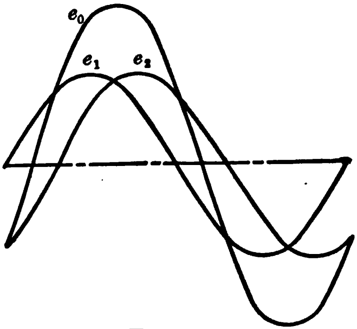


FIG. 95

(β) between e_0 and e_1 is given by $\tan^{-1}\dfrac{E_{m2}\sin\alpha}{E_{m1}+E_{m2}\cos\alpha}$ and the complete equation for the resultant e.m.f. is given by 89. It will be observed that any number of e.m.f.'s may be added in the same manner by the vector method. If the summation of the horizontal components is indicated by ΣH and the summation of the vertical components by ΣV, $e_0 = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}\,\sin\left(\omega t + \tan^{-1}\dfrac{\Sigma V}{\Sigma H}\right)$, the signs of the various components being fixed by the usual convention employed in the system of rectangular coördinates.

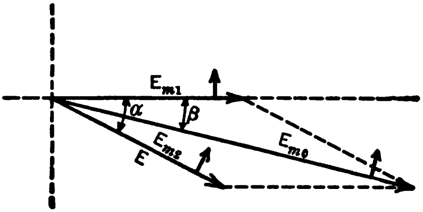


FIG. 96

Current established in Resistance by a Sinusoidal E.m.f. Since the current (i) established in a resistance (R) by an e.m.f. (e) from 14 is given by $i = \dfrac{e}{R}$ for any value of e, the current established in a resistance (R) in Fig. 97 by a sinusoidal e.m.f., $e = E_m \sin \omega t$, is given by

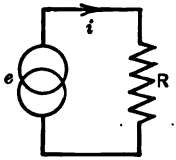


FIG. 97

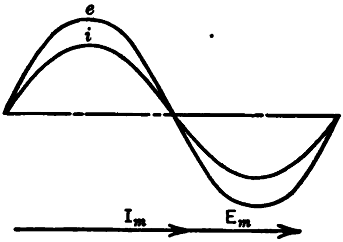


FIG. 98

90 $$i = \frac{E_m}{R}\sin \omega t \text{ amperes.}$$

The wave and vector diagrams for a circuit containing resistance only are shown in Fig. 98. It will be observed that the current and e.m.f. are in phase and the maximum current (I_m) equals the maximum e.m.f. (E_m) divided by the resistance (R).

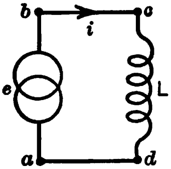


FIG. 99

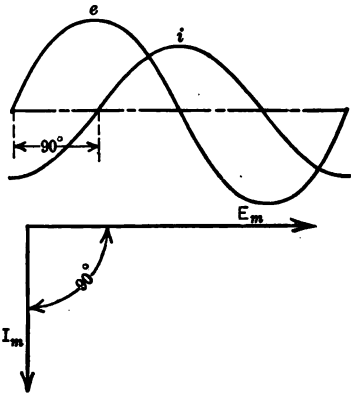


FIG. 100

Current established in Inductance by a Sinusoidal E.m.f. If an alternating current, $i_{cd} = I_m \sin \omega t$, is established in a circuit containing inductance only (Fig. 99) the e.m.f. induced in the inductance by the alternating current, from 54, is given by

$$\begin{aligned} e_{dc} &= L\frac{di_{cd}}{dt} \\ &= L\frac{d(I_m \sin \omega t)}{dt} \\ &= L\omega I_m \cos \omega t \\ &= L\omega I_m \sin (\omega t + 90°). \end{aligned}$$

An e.m.f., $e_{ab} = L\omega I_m \sin (\omega t + 90°)$, must then be impressed upon the circuit to maintain the current, $i_{cd} = I_m \sin \omega t$. If an e.m.f., $e = E_m \sin \omega t$, is impressed upon a circuit containing inductance only the current is given conversely by

91 $$i = \frac{E_m}{L\omega} \sin (\omega t - 90°) \text{ amperes.}$$

The wave and vector diagrams for such a circuit are given in Fig. 100. It will be observed that the current lags the e.m.f. by 90 degrees and the maximum current (I_m) equals the maximum e.m.f. (E_m) divided by the product of the inductance (L) and the angular velocity (ω).

The Capacitance of a Condenser. When two conducting materials are separated by an insulating material the combination is called a condenser (Fig. 101). When an e.m.f. is impressed upon a condenser containing a perfect insulator the current (i) flowing in the condenser equals the product of a constant (C) and the rate of change of the impressed e.m.f. $\left(\frac{de}{dt}\right)$ or

92 $$i = C\frac{de}{dt} \text{ amperes.}$$

The constant (C) is called the capacitance of the condenser and when the current is measured in amperes and the rate of change of the e.m.f. in volts per second the capacitance (C) is measured in farads. The microfarad, a smaller unit of capacitance, equals one millionth of a farad. The capacitance of a condenser is proportional to (1) the area (A) of each of the adjacent surfaces of the conducting materials, (2) the dielectric constant (k) of the insulation between the conducting materials and is inversely proportional to (3) the distance (d) between the conducting materials. The dielectric constants of various materials compared with air ($k = 1$) are given on page 185. When several condensers are connected in parallel the equivalent capacitance equals the sum of the individual capacitances or

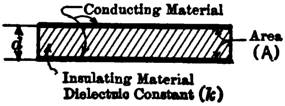


FIG. 101

93 $$C = C_1 + C_2 + C_3 \text{ farads.}$$

When several condensers are connected in series the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances, or

94 $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Current established in Capacitance by a Sinusoidal E.m.f. When a sinusoidal e.m.f., $e = E_m \sin \omega t$, is impressed upon a condenser of C farads capacitance (Fig. 102) the current, from 92, is given by

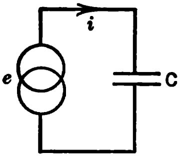
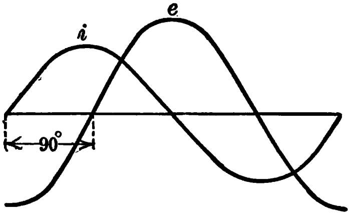


FIG. 102



$$i = C\frac{d(E_m \sin \omega t)}{dt} = C\omega E_m \cos \omega t$$

$$= C\omega E_m \sin(\omega t + 90°)$$

or

95 $$i = \frac{E_m}{\frac{1}{C\omega}} \sin(\omega t + 90°) \text{ amperes.}$$

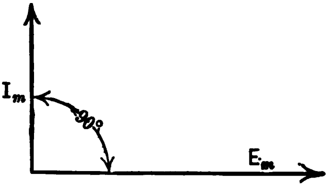


FIG. 103

The wave and vector diagrams for such a circuit are given in Fig. 103. It will be observed that the current leads the e.m.f. by 90 degrees and the maximum current equals the maximum e.m.f. (E_m) divided by the reciprocal of the product of the capacitance (C) and the angular velocity (ω).

Maximum E.m.f. required to maintain a Sinusoidal Current in Resistance, Inductance and Capacitance respectively. The maximum value (E_m) of the sinusoidal e.m.f. required to maintain a sinusoidal current of maximum value (I_m) in resistance, inductance and capacitance respectively may be summarized as follows:

Resistance (R)	$E_m = I_m R$	e and i in phase.
Inductance (L)	$E_m = I_m L\omega$	e leads i by 90 degrees.
Capacitance (C)	$E_m = \dfrac{I_m}{C\omega}$	e lags i by 90 degrees.

The Series Alternating Current Circuit. When an alternating current, $i = I_m \sin \omega t$, is established in a series circuit (Fig. 104) containing resistance, inductance and capacitance an e.m.f. of maximum value ($I_m R$) in phase with the current must be impressed upon the resistance, an e.m.f. of maximum value ($I_m L\omega$) leading the current by 90 degrees must be impressed upon the inductance, and an e.m.f. of maximum value $\left(\dfrac{I_m}{C\omega}\right)$ lagging the current by 90 degrees must be impressed upon the capacitance. The maximum value (E_m) of the e.m.f. that must be impressed upon the entire circuit, from Fig. 105, is given by $E_m = \sqrt{(I_m R)^2 + \left(I_m L\omega - \dfrac{I_m}{C\omega}\right)^2}$ or

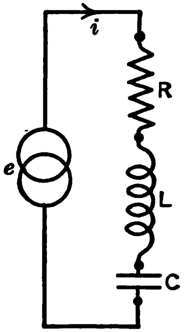


FIG. 104

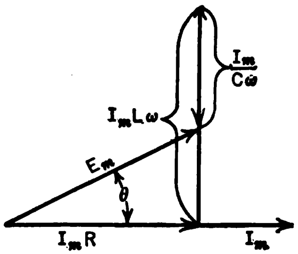


FIG. 105

96 $$E_m = I_m \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \text{ volts.}$$

Computations involving series connections are simplified by replacing $L\omega$, $\frac{1}{C\omega}$, $L\omega - \frac{1}{C\omega}$ and $\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ by the following symbols:

Symbol	Designation	Unit
$X_L = L\omega$	inductive reactance	ohm
$X_C = \frac{1}{C\omega}$	capacitive reactance	ohm
$X = L\omega - \frac{1}{C\omega}$	reactance	ohm
$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$	impedance	ohm

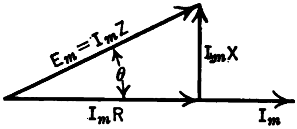


Fig. 106

In Fig. 106, the tangent of the phase angle (θ) between the e.m.f. and the current equals $\frac{I_m X}{I_m R}$ or

97 $$\theta = \tan^{-1}\frac{X}{R} \text{ degrees.}$$

The equation of the e.m.f. required to maintain a sinusoidal current, $i = I_m \sin \omega t$, in a series circuit is then given by

98 $$e = I_m Z \sin\left(\omega t + \tan^{-1}\frac{X}{R}\right) \text{ volts.}$$

When an e.m.f., $e = E_m \sin \omega t$, is impressed upon a series circuit the current is given conversely by

99 $$i = \frac{E_m}{Z} \sin\left(\omega t - \tan^{-1}\frac{X}{R}\right) \text{ amperes.}$$

When $L\omega = \frac{1}{C\omega}$ in a series circuit, $Z = R$ and the circuit is said to be in resonance.

Power delivered by a Source of Sinusoidal E.m.f. If the current established in a circuit by a sinusoidal e.m.f., $e = E_m \sin \omega t$, is $i = I_m \sin(\omega t - \theta)$ the power delivered to the circuit at any instant, from 3, is given by $p = (E_m \sin \omega t)(I_m \sin(\omega t - \theta)) = \frac{E_m I_m}{2}$ $[\cos\theta - \cos(2\,\omega t - \theta)]$ watts.

A plot of the instantaneous power is shown in Fig. 107. It will be noted that (1) the power wave is sinusoidal in form, (2) the frequency of the power wave is twice that of the e.m.f. or current, (3) the axis of the power wave (shown in dot and dash) will coincide with the axis of the e.m.f. and current waves only when $\theta = \pm 90$ degrees, and (4) the power during the interval (ab) and (cd) is negative; that is, the circuit during these intervals delivers power to the source of e.m.f. The net energy delivered to the circuit during one cycle of the e.m.f. equals the sum of the areas under the power wave during the intervals (bc) and (da') minus the sum of the areas under the power wave during the intervals (ab) and (cd). The average power (P), or the net energy delivered to the circuit during one cycle divided by the period (T), is then given by $P = \frac{1}{T}\int_0^T \frac{E_mI_m}{2}[\cos\theta - \cos(2\,\omega t - \theta)]\,dt$ or

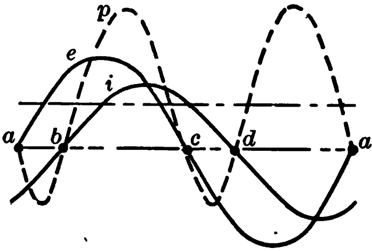


FIG. 107

100 $$P = \frac{E_mI_m\cos\theta}{2} \text{ watts.}$$

In any part of a circuit containing resistance (R) only, since $E_m = I_mR$ and θ equals zero, the average power absorbed by the resistance is given by

101 $$P = \frac{I_m^2R}{2} \text{ watts.}$$

Effective Value of an Alternating Current or E.m.f. When a direct current of I_{dc} amperes is established in a circuit of R ohms resistance electric energy is converted into thermal energy at a rate, $P = I_{dc}^2R$ watts. The average rate of energy conversion associated with a sinusoidal alternating current, $i = I_m \sin \omega t$, established in the same resistance, from 101, is $P = \frac{I_m^2R}{2}$ watts. For the same rate of energy conversion $I_{dc}^2R = \frac{I_m^2R}{2}$ and $I_{dc} = \frac{I_m}{\sqrt{2}}$. The direct current equivalent of an alternating current (the direct current which will convert the same amount of electric energy into thermal energy in the same resistance) is called the effective value of the alternating current. Hence the effective value (I) of a sinusoidal alternating current, $i = I_m \sin \omega t$, is given by

102 $$I = \frac{I_m}{\sqrt{2}} \text{ amperes.}$$

The direct e.m.f. equivalent of an alternating e.m.f. (the direct e.m.f. which will establish a direct current equal to the effective value of the current established by the alternating e.m.f. in the same resistance) is called the effective value of the alternating e.m.f. Since $E_{dc} = I_{dc}R = IR = \frac{I_m}{\sqrt{2}} R$ (for a sinusoidal current) and $E_m = I_mR$ then $E_{dc} \left(\text{substituting } \frac{E_m}{R} \text{ for } I_m\right) = \frac{E_m}{\sqrt{2}R} R = \frac{E_m}{\sqrt{2}}$. Hence the effective value (E) of a sinusoidal alternating e.m.f., $e = E_m \sin \omega t$, is given by

103 $$E = \frac{E_m}{\sqrt{2}} \text{ volts.}$$

Power, Power Factor and Kilovolt-Amperes. The power formula (100) for sinusoidal e.m.f. and current may be rewritten, $P = \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta$, and, since $\frac{E_m}{\sqrt{2}} = E$ and $\frac{I_m}{\sqrt{2}} = I$, the power in terms of the effective values of e.m.f. and current is given by

104 $$P = EI \cos \theta \text{ watts.}$$

The ratio of the power delivered to a circuit to the product of the effective e.m.f. and current, impressed upon and established in the circuit respectively, is called the power factor (p.f.) of the circuit. The power factor of any circuit is then given by

105 $$\text{p.f.} = \frac{P}{EI}$$

and, for a sinusoidal e.m.f. and current, by

106 $$\text{p.f.} = \cos \theta.$$

If each vector in Fig. 106 is divided by $\sqrt{2}$ the vectors will represent the effective values of the stated quantities as shown in Fig. 108. It will be noted that when all the electric energy delivered to a circuit is converted into thermal energy as in Fig. 104 the power factor of the circuit is given by $\cos\theta = \frac{IR}{IZ}$ or

107 $$\text{p.f.} = \frac{R}{Z}.$$

The component (I') of the current (I) in Fig. 108 in phase with E is called the energy component of the current and the component (I'') differing in phase with E by 90 degrees is called the wattless component of the current.

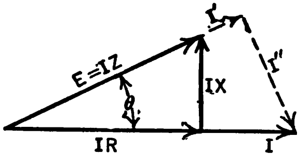


Fig. 108

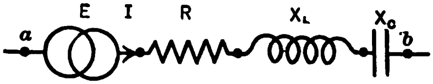


Fig. 109

The power converted in an alternating current circuit is given by one of the three expressions:

108 $$P = \frac{EI\cos\theta}{1000}\text{ kilowatts,}$$

109 $$P = \frac{VI\cos\theta}{1000}\text{ kilowatts,}$$

110 $$P = \frac{I^2R}{1000}\text{ kilowatts.}$$

The application of each formula is the same as that described under 3, 7 and 4.

The effective potential or voltage (V_{ab}) between two points (a and b in Fig. 109 for example) is given by

111 $$V_{ab} = E - IR - IX_L - IX_C\text{ volts,}$$

where E and I are the effective values of the e.m.f. and current and the subtraction is vectorial.

The kilovolt-amperes (KVA) delivered to any circuit is given by

112 $$KVA = \frac{VI}{1000}\text{ kilovolt-amperes.}$$

A Summary of the Alternating Current Series Circuit Relations. The following relations apply to Fig. 110:

$$R_{ad} = R_1 + R_2 \qquad X_{ad} = X_1 - X_2 + X_3 - X_4,$$
$$Z_{ab} = \sqrt{R_1^2 + X_1^2} \quad Z_{bc} = \sqrt{R_2^2 + X_2^2} \quad Z_{cd} = X_3 - X_4,$$

Z_{ad} does not $= Z_{ab} + Z_{bc} + Z_{cd}$ unless $\dfrac{X_{ab}}{R_{ab}} = \dfrac{X_{bc}}{R_{bc}} = \dfrac{X_{cd}}{R_{cd}}$ and each ratio has the same sign.

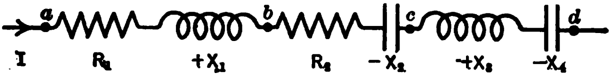


Fig. 110

$$Z_{ad} = \sqrt{R_{ad}^2 + X_{ad}^2},$$
$$V_{ab} = IZ_{ab} \qquad V_{bc} = IZ_{bc} \qquad V_{cd} = IZ_{cd},$$

V_{ad} does not $= V_{ab} + V_{bc} + V_{cd}$ unless $\dfrac{X_{ab}}{R_{ab}} = \dfrac{X_{bc}}{R_{bc}} = \dfrac{X_{cd}}{R_{cd}}$ and each ratio has the same sign.

$$V_{ad} = IZ_{ad}.$$

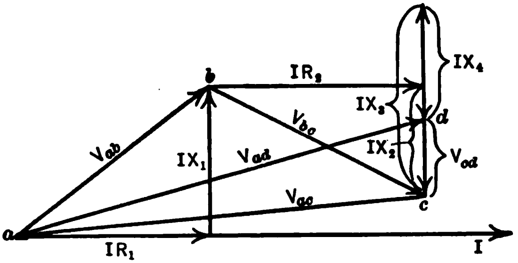


Fig. 111

The complete vector diagram for Fig. 110 is shown in Fig. 111, all vectors being located in accordance with the principles stated on page 79 and measured by effective values.

$$\cos\theta_{ab} = \frac{R_{ab}}{Z_{ab}}, \quad \cos\theta_{bc} = \frac{R_{bc}}{Z_{bc}}, \quad \cos\theta_{cd} = \frac{R_{cd}}{Z_{cd}}, \quad \cos\theta_{ad} = \frac{R_{ad}}{Z_{ad}}.$$

$$P_{ab} = V_{ab}I\cos\theta_{ab} = I^2R_{ab}, \qquad P_{bc} = V_{bc}I\cos\theta_{bc} = I^2R_{bc},$$
$$P_{cd} = V_{cd}I\cos\theta_{cd} = I^2R_{cd}, \qquad P_{ad} = V_{ad}I\cos\theta_{ad} = I^2R_{ad},$$
$$P_{ad} = P_{ab} + P_{bc} + P_{cd}.$$

The Parallel Connection in an Alternating Current Circuit. The vector diagram of a typical parallel connection (Fig. 112) is shown in Fig. 113, all vectors being referred to the effective potential (V_{mn}) between m and n. In Fig. 112, $I_{ab} = \frac{V_{mn}}{Z_1}$, $I_{cd} = \frac{V_{mn}}{Z_2}$, $\cos\theta_{ab} = \frac{R_1}{Z_1}$ and $\cos\theta_{cd} = \frac{R_2}{Z_2}$. I_{mn} equals the vector sum of I_{ab} and I_{cd} as shown in Fig. 113. The magnitude and phase relation of I_{mn} may be determined from its horizontal and vertical components as explained in the addition of e.m.f.'s on page 76. The horizontal component of I_{mn} is given by

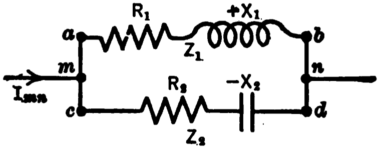


FIG. 112

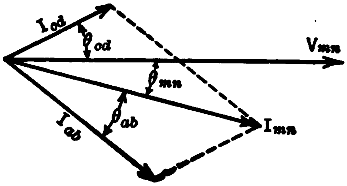


FIG. 113

$$\Sigma H = \frac{V_{mn}}{Z_1} \cdot \frac{R_1}{Z_1} + \frac{V_{mn}}{Z_2} \cdot \frac{R_2}{Z_2} = V_{mn}\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2}\right).$$

The vertical component of I_{mn} is given by

$$\Sigma V = -\frac{V_{mn}}{Z_1} \cdot \frac{X_1}{Z_1} + \frac{V_{mn}}{Z_2} \cdot \frac{X_2}{Z_2} = -V_{mn}\left(\frac{X_1}{Z_1^2} - \frac{X_2}{Z_2^2}\right).$$

Since $I_{mn} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$,

$$I_{mn} = \sqrt{\left[V_{mn}\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2}\right)\right]^2 + \left[-V_{mn}\left(\frac{X_1}{Z_1^2} - \frac{X_2}{Z_2^2}\right)\right]^2} \text{ or}$$

113 $$I_{mn} = V_{mn}\sqrt{\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2}\right)^2 + \left(\frac{X_1}{Z_1^2} - \frac{X_2}{Z_2^2}\right)^2} \text{ amperes.}$$

The phase angle (θ_{mn}) equals $\tan^{-1}\frac{\Sigma V}{\Sigma H}$ or

114 $$\theta_{mn} = \tan^{-1} - \left(\frac{\frac{X_1}{Z_1^2} - \frac{X_2}{Z_2^2}}{\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2}}\right) \text{ degrees.}$$

Conductance, Susceptance and Admittance. Computations involving parallel connections are simplified somewhat by replacing $\frac{R}{Z^2}$, $\frac{X}{Z^2}$, and $\sqrt{\left(\frac{R}{Z^2}\right)^2 + \left(\frac{X}{Z^2}\right)^2}$ by the following symbols:

Symbol	**Designation**	**Unit**
$G = \frac{R}{Z^2}$	conductance	mho
$B = \frac{X}{Z^2}$	susceptance	mho
$Y = \sqrt{\left(\frac{R}{Z^2}\right)^2 + \left(\frac{X}{Z^2}\right)^2}$	admittance	mho

Formula 113 may then be written,

$$I_{mn} = V_{mn} \sqrt{(G_1 + G_2)^2 + (B_1 + B_2)^2} = V_{mn} \sqrt{(G_{mn})^2 + (B_{mn})^2} \text{ or}$$

115 $$I_{mn} = V_{mn}\, Y_{mn} \text{ amperes.}$$

The phase angle (θ_{mn}) is given by

116 $$\theta_{mn} = \tan^{-1} - \left(\frac{B_{mn}}{G_{mn}}\right) \text{ degrees.}$$

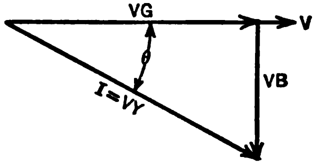


FIG. 114

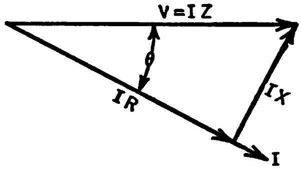


FIG. 115

The vector diagram for any part of an alternating current circuit expressed in terms of conductance, susceptance and admittance is shown in Fig. 114 (the admittance diagram) and the vector diagram for the same part of the circuit expressed in terms of resistance, reactance and impedance is shown in Fig. 115 (the impedance diagram). Since $I = VY = \frac{V}{Z}$, it follows that

117 $$Y = \frac{1}{Z} \text{ mhos or } Z = \frac{1}{Y} \text{ ohms.}$$

In Fig. 114, $\cos\theta = \frac{G}{Y}$ and in Fig. 115, $\cos\theta = \frac{R}{Z}$. Then $\frac{R}{Z} = \frac{G}{Y}$ and since $\frac{R}{Z} = RY$,

118 $$R = \frac{G}{Y^2} \text{ ohms.}$$

In Fig. 114, $\sin\theta = \frac{B}{Y}$ and in Fig. 115, $\sin\theta = \frac{X}{Z}$. Then $\frac{X}{Z} = \frac{B}{Y}$ and since $\frac{X}{Z} = XY$, 119 $$X = \frac{B}{Y^2} \text{ ohms.}$$

When the resultant susceptance (B) of a parallel connection equals zero, $Y = G$ and the parallel connection is said to be in resonance.

A Summary of the Alternating Current Parallel Connection Relations. The following relations apply to Fig. 112:

$$G_{ab} = \frac{R_1}{Z_1^2} \qquad G_{cd} = \frac{R_2}{Z_2^2} \qquad G_{mn} = G_{ab} + G_{cd}$$

$$B_{ab} = \frac{X_1}{Z_1^2} \qquad B_{cd} = \frac{-X_2}{Z_2^2} \qquad B_{mn} = B_{ab} + B_{cd}.$$

It should be noted that the summation of conductances is arithmetic while the summation of susceptances is algebraic since the reactance is positive for inductive reactance and negative for capacitive reactance.

$$Y_{ab} = \sqrt{G_{ab}^2 + B_{ab}^2} \qquad Y_{cd} = \sqrt{G_{cd}^2 + B_{cd}^2}.$$

Y_{mn} does not $= Y_{ab} + Y_{cd}$ unless $\frac{B_{ab}}{G_{ab}} = \frac{B_{cd}}{G_{cd}}$ and each ratio has the same sign.

$$Y_{mn} = \sqrt{(G_{mn})^2 + (B_{mn})^2}.$$

$$I_{ab} = \frac{V_{mn}}{Z_1} = V_{mn}Y_{ab} \qquad I_{cd} = \frac{V_{mn}}{Z_2} = V_{mn}Y_{cd}.$$

I_{mn} does not $= I_{ab} + I_{cd}$ unless $\frac{B_{ab}}{G_{ab}} = \frac{B_{cd}}{G_{cd}}$ and each ratio has the same sign.

$$I_{mn} = \frac{V_{mn}}{Z_{mn}} = V_{mn}Y_{mn},$$

$$\cos\theta_{ab} = \frac{R_1}{Z_1} = \frac{G_{ab}}{Y_{ab}} \qquad \cos\theta_{cd} = \frac{R_2}{Z_2} = \frac{G_{cd}}{Y_{cd}},$$

$$\cos\theta_{mn} = \frac{R_{mn}}{Z_{mn}} = \frac{G_{mn}}{Y_{mn}},$$

$$R_{mn} = \frac{G_{mn}}{Y_{mn}^2} \qquad X_{mn} = \frac{B_{mn}}{Y_{mn}^2}.$$

The Series-Parallel Alternating Current Circuit. When an alternating current circuit contains parallel connections the equivalent resistance and reactance of each parallel connection must be determined and the equivalent series connection substituted for each parallel connection. In Fig. 116, for example, $G_{bc} = G_2 + G_3$ and $B_{bc} = B_2 + B_3$. Then $R_{bc} = \frac{G_{bc}}{G_{bc}^2 + B_{bc}^2}$ and $X_{bc} = \frac{B_{bc}}{G_{bc}^2 + B_{bc}^2}$.

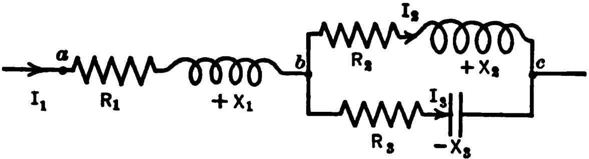


FIG. 116

$R_{ac} = R_1 + R_{bc}$, $X_{ac} = X_1 + X_{bc}$, and $Z_{ac} = \sqrt{R_{ac}^2 + X_{ac}^2}$. $I_1 = I_{ac} = I_{bc} = \frac{V_{ac}}{Z_{ac}}$. $Z_{bc} = \sqrt{R_{bc}^2 + X_{bc}^2}$ and $V_{bc} = I_{bc}Z_{bc}$. Then $I_2 = \frac{V_{bc}}{Z_2}$ and $I_3 = \frac{V_{bc}}{Z_3}$.

CHAPTER VII

ALTERNATING CURRENT MEASUREMENTS

Measurement of Effective Current and Potential. Since the power converted into heat by a direct current is I_{dc}^2R and the average power converted into heat by an alternating current is $\frac{1}{T}\int_0^T i^2R\,dt$ the direct current equivalent or the effective value (I) of an alternating current is given by equating the two expressions and solving for I_{dc}. The effective value (I) of an alternating current is then given by

120 $$I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \text{ amperes.}$$

In any instrument which indicates the effective value of an alternating current the force acting upon the moving element must then be proportional to i^2, the square of the instantaneous current, and the moving element must take a position corresponding to the average value of the squared currents during one cycle; that is, the deflection must be proportional to $\frac{1}{T}\int_0^T i^2\,dt$. The deflection being proportional to the average square of the current the scale divisions on such instruments will not be as uniform as on the direct current instrument described on page 67.

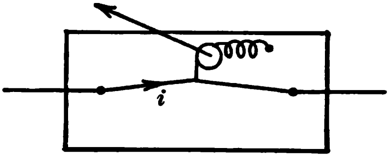


Fig. 117

The Hot Wire Ammeter. A platinum-silver wire is stretched between two insulated points on a bronze plate of the same coefficient of expansion, as shown in Fig. 117. Since the heating effect of an electric current (i) is proportional to i^2R the temperature of the platinum-silver wire when conducting an alternating current will be proportional to $\frac{1}{T}\int_0^T i^2\,dt$, the mass of the wire being sufficient to maintain it at a constant temperature while the current passes through its cyclical values. The length of the wire, being proportional to its temperature, is then proportional to $\frac{1}{T}\int_0^T i^2\,dt$. A string attached to the center of the wire is wound around a small pulley and is held taut by a spring so that any change in the length of the wire is indicated by the proportional angular motion of a pointer attached to the pulley.

This instrument, called a hot wire ammeter, will then indicate the effective value of an alternating current. The instrument is seldom used for ordinary measurements of current since it must be calibrated frequently and is somewhat sluggish in action. It is sometimes employed in circuits in which the effective value of the current varies rapidly (making use of its sluggish characteristic) and is applied extensively to the measurement of alternating currents of high frequency (as in radio telegraph and telephone circuits). It is used frequently as a comparison instrument in the calibration of alternating current instruments, the deflection of the hot wire ammeter due to an unknown alternating current being later interpreted by a known direct current producing the same deflection. A hot wire voltmeter may be constructed on the same principle (a high resistance is connected in series with the platinum-silver wire) but the instrument possesses no particular advantages.

The Electrodynamometer Ammeter and Voltmeter. A pivoted movable coil controlled by spiral springs is placed within a fixed coil as shown in Fig. 118. The flux density (B) due to the fixed coil, from 37, is proportional to the current (i) and since the movable coil is connected in series with the fixed coil and carries the same current (i) the force acting on the movable coil, from 36, is proportional to iB or i^2. The movable coil then takes up a

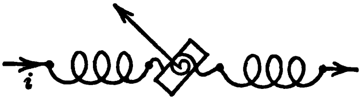


FIG. 118

position proportional to $\frac{1}{T}\int_0^T i^2\,dt$ by reason of its inertia. The instrument constructed as shown in Fig. 118 and called an electrodynamometer ammeter is adapted to the direct measurement of an alternating current not exceeding 0.75 ampere in effective value, the current being limited by the current capacity of the spiral springs through which electrical connections are made to the moving coil. Electrodynamometer ammeters designed to measure currents up to 10 amperes are sometimes made (the movable coil is then shunted) but such instruments require careful adjustment for accurate measurement at all frequencies and are not generally used.

In the electrodynamometer voltmeter a high non-inductive resistance is connected in series with the unit shown in Fig. 118, the resistances of the fixed and movable coils also being higher than in the electrodynamometer ammeter. The deflection of the voltmeter is then proportional to the effective potential between its terminals since the effective current which produces the deflection equals the quotient of the effective potential and the resistance of the instrument. The electrodynamometer principle is well adapted to voltmeter construction since the current conducted by the spiral springs in a voltmeter does not exceed a few hundredths of an ampere except in the low range instruments (0.5 ampere in a one volt voltmeter). Most alternating current voltmeters are therefore of the electrodynamometer type.

The Iron Vane Ammeter and Voltmeter. If an iron vane is placed within an inclined coil conducting an alternating current as shown in Fig. 119, the force acting on the iron vane at any instant is proportional to the square of the flux density due to the current in the coil (see page 25). Since the flux density due to the current is proportional to the current the force acting on the iron vane is proportional to the square of the current. The pointer attached to the iron vane will then take up a position proportional to $\frac{1}{T}\int_0^T i^2\,dt$ by reason of the inertia of the moving element. A spiral spring attached to the moving element provides the reactive torque

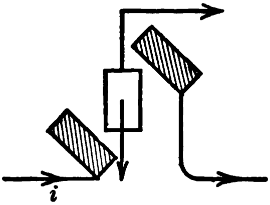


FIG. 119

as in instruments previously described. The iron vane principle is well adapted to ammeter construction since the moving element carries no current. Most alternating current ammeters are therefore of the iron vane type.

In the iron vane voltmeter a high non-inductive resistance is connected in series with the inclined coil which is also of higher resistance than the coil used in the iron vane ammeter. The deflection of the pointer, as in the electrodynamometer voltmeter, is then proportional to the effective potential between the terminals of the instrument. The iron vane voltmeter is seldom used for accurate measurements, however, because of the superior qualities of the electrodynamometer type.

The Electrodynamometer Wattmeter. Since the power factor of an alternating current circuit is seldom known the power cannot generally be determined from voltmeter and ammeter readings. An instrument measuring the power directly, called a wattmeter, is therefore indispensable. In the electrodynamometer wattmeter (Fig. 120) the fixed coil is connected in series with the line (or conducts a certain portion of the line current) and the movable coil in series with a high non-inductive resistance is connected across the line. The instantaneous current (i_f) in the fixed coil then equals (or is proportional to) the instantaneous line current (i_l), and the instantaneous current (i_m) in the movable coil is proportional to the instantaneous potential (v_l) between the line wires. The instantaneous flux density due to the fixed coil is then proportional to i_l and, from 36, the force acting upon the movable coil is proportional to v_li_l or the instantaneous power supplied through the line to which the wattmeter is connected. The deflection of a pointer attached to the movable coil by reason of its inertia will then be proportional to the average power or $V_lI_l \cos \theta$.

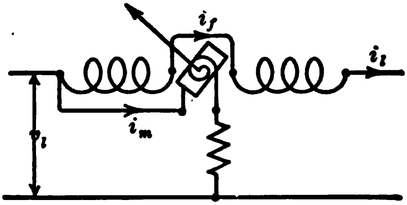


FIG. 120

The Induction Watt-hour Meter. While the Thomson watt-hour meter described on page 69 may be employed with slight alteration for the measurement of energy in an alternating current circuit the induction watt-hour meter shown in Fig. 121 is better adapted for the purpose. A revolving disc (D) is mounted between a potential coil (P) and the current coils (CC). The flux (ϕ_c) permeating the disc due to the current coils is proportional to and in phase with i_l. The instantaneous current (i_p) flowing in the potential coil (P) is proportional to the instantaneous line potential

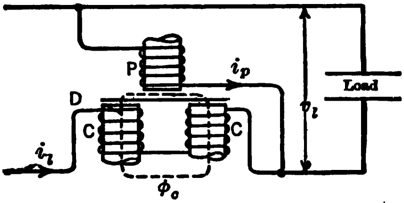


FIG. 121

(v_l) but lags it by nearly 90 degrees by reason of the highly inductive character of the potential coil. The flux (ϕ_p) due to the potential coil current is proportional to and in phase with the potential coil current (i_p). The e.m.f. (e_d) generated in the disc (D) by the alternating flux (ϕ_p), from 53, is proportional to ϕ_p and leads it by 90 degrees. The current (i_d) established in the disc by e_d is then proportional to and in phase with v_l.

The force acting upon the disc at any instant due to the reaction between i_d and ϕ_c, from 36, is then proportional to the product of v_l and i_l, or the instantaneous power supplied to the load, and the average torque acting upon the disc is proportional to the average power (P) supplied to the load or $V_lI_l \cos \theta$. The disc also rotates between the poles of two or more permanent magnets and the reactive torque (as described on page 69) acting on the disc is therefore proportional to the speed (S) of the disc. Then $P = KS$ and the energy (W) supplied to the load in any time (t) is given by $W = Pt = KSt$ or $W = KR$ where R is the number of revolutions of the disc in the time (t). A series of dials (similar to those shown in Fig. 84) geared to the disc shaft will then indicate the number of kilowatt-hours supplied to the load in a given time.

Instrument Transformers. The potential coils of most alternating current instruments are designed for direct connection to circuits of 150 volts effective potential between line wires (300, 600 and 750 volt instruments are also available) and the current coils are usually designed to conduct not more than 5 amperes effective current. High non-inductive resistances, called extension coils or multipliers, may be connected in series with potential coils when the effective line potential exceeds the range of the instrument. The potential reading of the instrument is then multiplied by the ratio $\left(\frac{R_p + R_m}{R_p}\right)$ where R_p is the resistance of the potential coil and R_m is the resistance of the multiplier. This use of the multiplier to increase the range of an instrument is restricted, however, to instruments in which the inductance of the potential coil is negligible.

The shunting of current coils to increase the range of an instrument is not as desirable in alternating as in direct current instruments since the division of current in the alternating current instrument between the current coil and the shunt will depend upon the frequency of the current unless the inductance of each connection is made negligible. The range of alternating current instruments is extended most conveniently and accurately by connecting potential and current transformers to the potential and current coils as shown in Fig. 122. The line current (I_l) then equals $T_c I_c$ and the line potential (V_l) equals $T_p V_p$, where T_c and T_p are the ratios of transformation of the current and potential transformers respectively. The principle of the transformer is discussed on page 128.

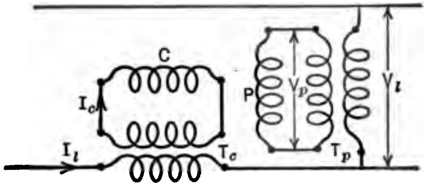


FIG. 122

The Frequency Meter. The measurement of frequency is often accomplished by an instrument in which a series of steel reeds (Fig. 123) of definite successive frequencies of vibration are placed in the alternating magnetic field produced by a coil of high resistance wire

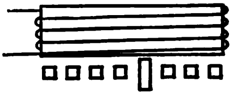


FIG. 123

connected across the line. The frequency of the alternating magnetic flux will then be the same as that of the line potential and the reed of the same frequency will be set in vibration. The end of each reed is painted white and the frequency is indicated by the white blur produced opposite a scale indicating the frequency of each reed. Frequency meters are also made in which the coil currents vary with the change of reactance due to a change in frequency.

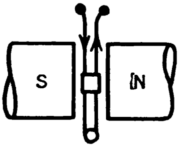


FIG. 124

The Oscillograph. The wave form of an alternating current may be photographed with an instrument, called an oscillograph, which contains a vibrator, an intense concentrated source of light, and a photographic film attached to the surface of a rotating drum. The vibrator consists of a doubled fine strip of copper held taut between the poles of a powerful electromagnet as shown in Fig. 124. A small mirror cemented to the copper strip reflects a spot of light onto the film covering the rotating drum as shown in Fig. 125. When an alternating current is sent through the vibrator the angular twist of the vibrator at any instant is proportional to the instantaneous current.

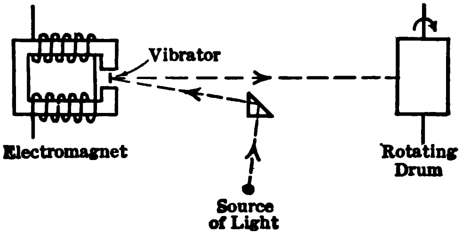


FIG. 125

A horizontal line of light would then be seen on the surface of the drum parallel to its axis, the length of the line (to scale) being equal to twice the maximum value of the vibrator current. Since the drum is rotated at constant speed the sensitized film on its surface will record the successive positions of the spot of light and when developed will reveal the wave form of the vibrator current. If light is reflected from two vibrators onto the drum, one vibrator being connected across the line in series with a non-inductive resistance and the other vibrator connected in series with the line (usually shunted by a low resistance), the simultaneous wave forms of the line potential and the line current including their phase relation may be photographed. If the surface of the rotating drum is covered with several plane mirrors the vibrating spots of light may again be reflected upward to a plate of ground glass The wave forms may then be seen and traced upon thin paper.

CHAPTER VIII

THREE-PHASE ALTERNATING CURRENTS

The Generation of Three-phase E.m.f.'s. A single-phase generator has one armature winding (represented by the single turn *xa* in Fig. 126) in which an e.m.f., $e_{xa} = E_m \sin \omega t$, is generated. A three-phase generator has three armature windings (represented by the single turns *xa*, *yb* and *zc* in Fig. 127) located 120 degrees apart and in which the respective generated e.m.f.'s differ in phase by 120 degrees. The three generated e.m.f.'s may be represented mathematically by $e_{xa} = E_m \sin \omega t$, $e_{yb} = E_m \sin (\omega t - 120°)$ and $e_{zc} = E_m \sin (\omega t - 240°)$. The same e.m.f.'s may be represented by a wave diagram (Fig. 128) and by a vector diagram (Fig. 129).

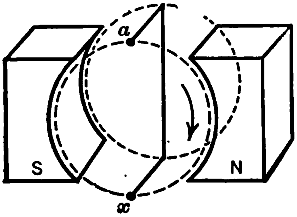


FIG. 126

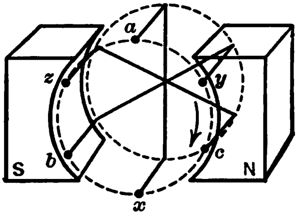


FIG. 127

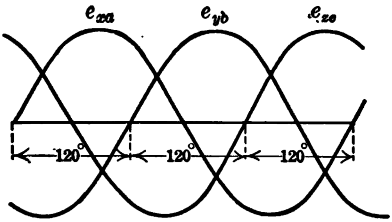


FIG. 128

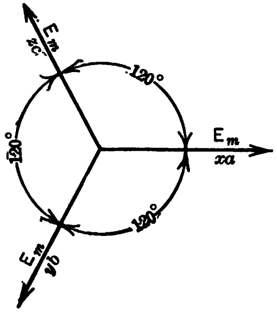


FIG. 129

The Three-phase Y Connection. If the three armature windings of a three-phase generator are connected independently by six slip rings to three identical loads, $x'a'$, $y'b'$ and $z'c'$ as shown in Fig. 130 the line currents i_{xa}, i_{yb} and i_{zc} will also differ in phase by 120 degrees and each current will differ in phase with its e.m.f. by the same angle (θ). The load is then said to be balanced and a balanced load is assumed throughout the chapter. The currents, referred to i_{xa}, may be represented mathematically by $i_{xa} = I_m \sin \omega t$, $i_{yb} = I_m \sin (\omega t - 120°)$ and $i_{zc} = I_m \sin (\omega t - 240°)$. The same currents may be represented by a wave diagram (Fig. 131) and by a vector diagram (Fig. 132).

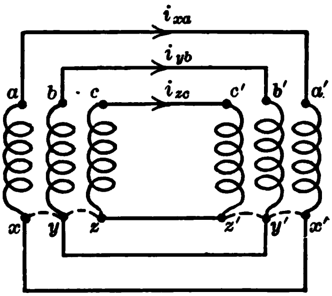


FIG. 130

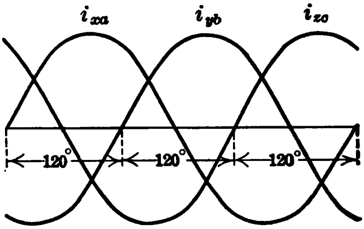


FIG. 131

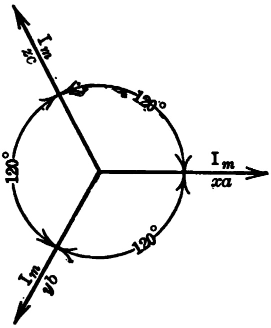


FIG. 132

It will be noted (Fig. 131 for example) that $i_{xa} + i_{yb} + i_{zc} = 0$; that is, in Fig. 130, the sum of the currents at any instant flowing to the right between xx', yy' and zz' equals the sum of the currents at the same instant flowing to the left. The line wires xx', yy' and zz' are therefore unnecessary. If these line wires are removed and the terminals xyz and $x'y'z'$ are connected as shown by the dotted lines the system may be operated with one-half the weight of line wire required in the previous arrangement.

The latter connection, called a Y connection, is usually represented by the connection diagram shown in Fig. 133, substituting *oa* for *xa*, *ob* for *yb*, *oc* for *zc*, etc. The connection point (*o*), common to all three windings, is called the neutral point of the generator; *o′* is the neutral point of the load. E_{oa}, E_{ob} and E_{oc} are called the effective phase e.m.f.'s (E_p) and E_{ab}, E_{bc} and E_{ca} are called the effective line e.m.f.'s (E_l) of the generator. The vector diagram of the phase e.m.f.'s and line e.m.f.'s of a Y-connected three-phase generator is usually constructed as shown in Fig. 134, although any axis of reference may be assumed. The line e.m.f. (E_{ab}) at the generator equals the vector sum of E_{ao} and E_{ob}. From Fig. 134 it will be seen that $E_{ab} = 2\ E_{ob} \cos 30°$ or $E_{ab} = \sqrt{3}\ E_{ob}$. Since the effective phase e.m.f.'s are equal and the effective line e.m.f.'s are equal it follows that in the Y connection.

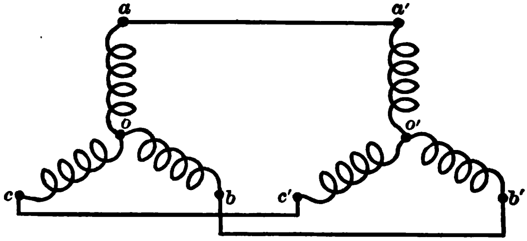


FIG. 133

121 $$E_l = \sqrt{3}E_p \text{ volts.}$$

The effective phase currents (I_{oa}, I_{ob} and I_{oc}) are equal and the effective line currents ($I_{aa'}$, $I_{bb'}$ and $I_{cc'}$) are equal. Since I_{oa} equals $I_{aa'}$, for example, it follows that in the *Y* connection the effective line current (I_l) equals the effective phase current (I_p) or

122 $$I_l = I_p \text{ amperes.}$$

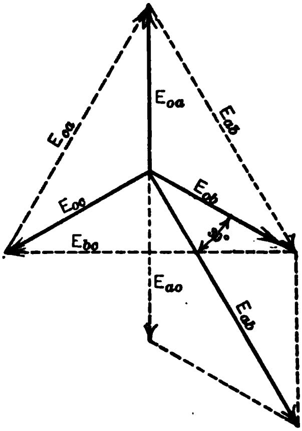


FIG. 134

The power (P_p) converted by each phase of a Y-connected generator, from 104, is given by $P_p = E_pI_p \cos \theta_p$ where θ_p is the phase angle between E_p and I_p. The total power (P) converted by the generator is then given by $P = 3\,E_pI_p \cos\theta_p$. Substituting $\frac{E_l}{\sqrt{3}}$ for E_p and I_l for I_p the total power is given by

$$P = \sqrt{3}\,E_lI_l \cos\theta_p \text{ watts.} \tag{123}$$

It should be noted that $\cos\theta_p$, the power factor of the three-phase system, is not the cosine of the phase angle between the line e.m.f. and the line current but is the cosine of the phase angle between the phase e.m.f. and the phase current.

The Three-phase Δ Connection. The three line e.m.f.'s of a Y-connected generator are equal and differ in phase by 120 degrees as explained in the previous paragraph. If the three armature windings of a three-phase generator (*ab*, *bc* and *ca*) are connected as shown in Fig. 135 the line e.m.f.'s will also be equal and differ in phase by 120 degrees. This is called a Δ connection. The load in Figs. 133 and 135 may be either Y- or Δ-connected. The effective phase currents (I_{ab}, I_{bc} and I_{ca}) of a Δ-connected three-

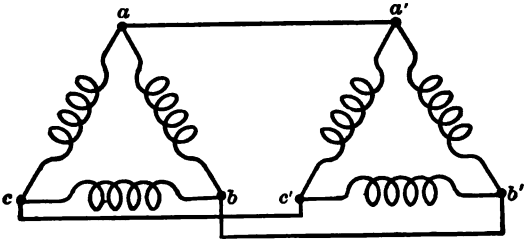


FIG. 135

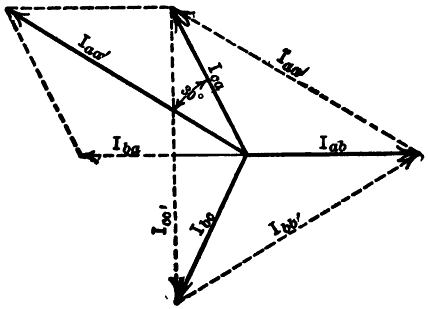


FIG. 136

phase generator supplying power to a balanced three-phase load will be equal and differ in phase by 120 degrees as shown in Fig. 136. Since the respective line e.m.f.'s and phase e.m.f.'s are identical it follows that in the Δ connection

124 $$E_l = E_p \text{ volts.}$$

The effective line current $I_{aa'}$ (for example) equals the vector sum of the effective phase currents I_{ca} and I_{ba}. Then in Fig. 136, $I_{aa'} = 2\,I_{ca} \cos 30°$ or $I_{aa'} = \sqrt{3}\,I_{ca}$. Since the effective phase currents are equal and the effective line currents are equal it follows that in the Δ connection

125 $$I_l = \sqrt{3}\,I_p \text{ amperes.}$$

The power (P_p) converted by each phase of a Δ-connected generator, from 104, is given by $P_p = E_pI_p \cos \theta_p$, where θ_p is the phase angle between E_p and I_p. The total power (P) converted by the generator is then given by $P = 3\,E_pI_p \cos \theta_p$. Substituting E_l for E_p and $\dfrac{I_l}{\sqrt{3}}$ for I_p the total power is given by 123.

The kilovolt-amperes supplied by any three-phase generator is given by

126 $$KVA = \frac{\sqrt{3}\,E_lI_l}{1000} \text{ kilovolt-amperes.}$$

The Three-phase V and T Connections. Three-phase line e.m.f.'s may be generated in an armature containing two windings located 120 degrees apart and connected as shown in Fig. 137.

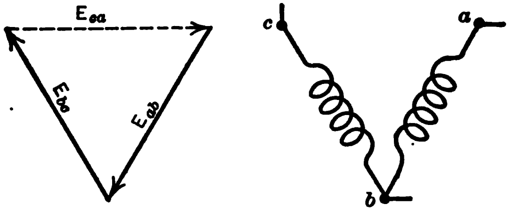


FIG. 137

This is called a V connection. It will be noted that a Δ connection with one winding disconnected constitutes a V connection. If two windings located 90 degrees apart are connected as shown in

Fig. 138 and $E_{oa} = \frac{\sqrt{3}}{2} E_{bc}$, the line e.m.f.'s will be equal and differ in phase by 120 degrees. This is called a T connection.

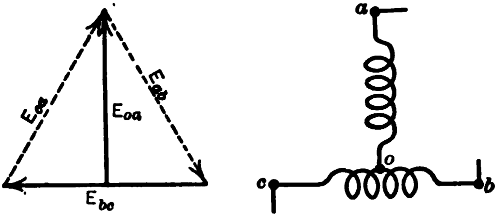


FIG. 138.

Other Polyphase Connections. Two windings located 90 degrees apart in an armature (Fig. 139) will generate e.m.f.'s differing in phase by 90 degrees. The generator is then called a two-phase generator; the line e.m.f.'s E_{oa} and E_{ob} in Fig. 139 are equal but the line e.m.f. E_{ab} equals $\sqrt{2}\, E_{oa}$ or $\sqrt{2}\, E_{ob}$. If four windings located 90 degrees apart are connected as shown in Fig. 140 the generator is called a four-phase (quarter-phase) generator. In the six-phase generator the six armature windings are located 60 degrees

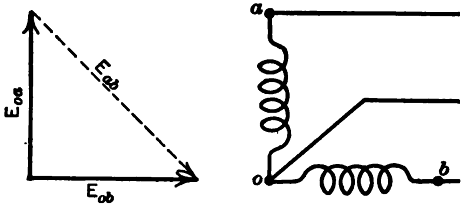


FIG. 139

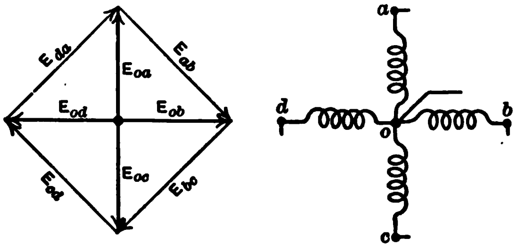


FIG. 140

apart as shown in Fig. 141. Most modern alternating current generators are Y-connected three-phase generators. Two-phase and four-phase generators are employed occasionally in special

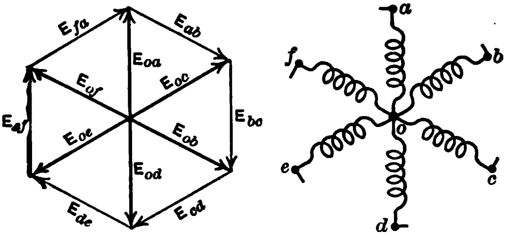


FIG. 141

cases and six-phase e.m.f.'s are frequently impressed upon synchronous converters (see page 121).

Measurement of Three-phase Power by Three Wattmeters. In the three-wattmeter method the current and potential coils are

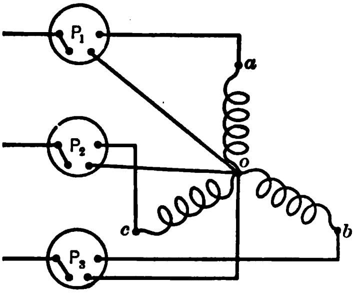


FIG. 142

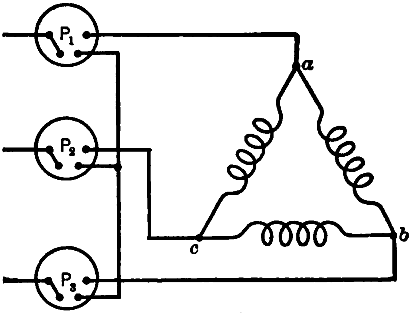


FIG. 143

connected as shown in Fig. 142 for a Y-connected load and as shown in Fig. 143 for a Δ-connected load. In either case the total power (P) is given by

$$P = P_1 + P_2 + P_3 \text{ watts.} \tag{127}$$

If the load is balanced the total power may be determined from the reading of a single wattmeter since $P = 3\ P_1$ or three times the power indicated by one wattmeter. In the Δ-connected load the potential coil circuit of the single wattmeter and two non-inductive circuits of equal resistance are connected in Y between *a*, *b* and *c* to furnish an artificial neutral.

The Two Wattmeter Method. The instantaneous power (p) supplied to the load shown in Fig. 144, from 7, is given by $p = v_{oa}i_{oa} + v_{ob}i_{ob} + v_{oc}i_{oc}$. Since $i_{oc} = -i_{oa} - i_{ob}$, $p = v_{oa}i_{oa} + v_{ob}i_{ob} - v_{oc}i_{oa} - v_{oc}i_{ob} = (v_{oa} - v_{oc})i_{oa} + (v_{ob} - v_{oc})i_{ob} = v_{ca}i_{oa} + v_{cb}i_{ob}$. If two wattmeters are connected as shown in Fig. 144, P_1 will equal the average value of $v_{ca}i_{oa}$ and P_2 will equal the average value of $v_{cb}i_{ob}$. The effective phase potentials, line potentials and phase currents affecting the wattmeter readings in Fig. 144 are shown in Fig. 145. The average value of $v_{ca}i_{oa}$ is given by $V_{ca}I_{oa}$ cos $(30° - \theta)$. Hence the power indicated by wattmeter No. 1 is given by

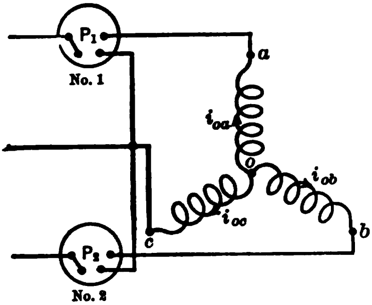


Fig. 144

128 $$P_1 = V_lI_l \cos(30° - \theta) \text{ watts.}$$

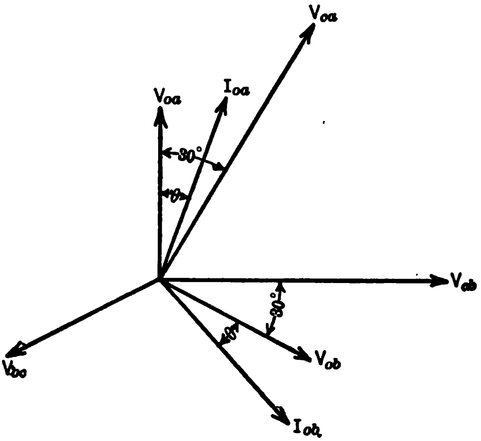


Fig. 145

The average value of $v_{cb}i_{ob}$ is given by $V_{cb}I_{ob}$ cos $(30° + \theta)$. Hence the power indicated by wattmeter No. 2 is given by

129 $$P_2 = V_lI_l \cos(30° + \theta) \text{ watts.}$$

When the phase angle (θ) is not greater than 60 degrees both wattmeters will read "on scale" and the total power is given by

130 $$P = P_1 + P_2 \text{ watts.}$$

When the phase angle (θ) is greater than 60 degrees (lagging current) wattmeter No. 2 will read "off scale." To obtain a reading "on scale" the connections to its current coil must be reversed and since the resultant reading is negative the total power is given by

131 $$P = P_1 - P_2 \text{ watts.}$$

If the phase angle (θ) is unknown the potential coil connection of wattmeter No. 1 (for example) to line c should be broken and then reconnected to line b. If wattmeter No. 1 still reads "on scale," $P = P_1 + P_2$. If wattmeter No. 1 reads "off scale" after the reconnection, $P = P_1 - P_2$.

Since $P_1 = V_l I_l \cos (30° - \theta)$ and $P_2 = V_l I_l \cos (30° + \theta)$ then

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{V_l I_l [\cos (30° - \theta) - \cos (30° + \theta)]}{V_l I_l [\cos (30° - \theta) + \cos (30° + \theta)]}$$

$$= \frac{2 \sin 30° \sin \theta}{2 \cos 30° \cos \theta} = \tan 30° \tan \theta = \frac{\tan \theta}{\sqrt{3}} \quad \text{and}$$

132 $$\tan \theta = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}.$$

CHAPTER IX

THE ALTERNATING CURRENT TRANSMISSION LINE

Choice of System and Potential. The power transmitted by a single-phase transmission line, Fig. 146 (*a*), is given by $P = VI \cos \theta$ and the power transmitted by a three-phase transmission line, Fig. 146 (*b*), is given by $P = \sqrt{3}\, VI' \cos \theta$. For the same power transmitted by each line $VI \cos \theta = \sqrt{3}\, VI' \cos \theta$ and $I' = \frac{I}{\sqrt{3}}$. The power converted into heat in the single-phase transmission line is given by $P = 2\, I^2R$, where R is the resistance of each line wire. The power converted into heat in the three-phase transmission line is given by $P = 3\, I'^2R'$, where R' is the resistance of each line wire. Substituting $\frac{I}{\sqrt{3}}$ for I', $P = 3\frac{I^2}{3} R' = I^2R'$. For the same power loss in each transmission line $2\, I^2R = I^2R'$ and $R' = 2\, R$. The weight (W') of each three-phase line wire then equals one-half the weight (W) of each single-phase line wire or $W' = \frac{W}{2}$. The ratio of the total weight of the three-phase line to the total weight of the single-phase line is then $\frac{3\,W}{2} \cdot \frac{1}{2\,W}$ or 0.75. The use of a three-phase transmission line is then favored by a saving of 25 per cent of the weight of wire required in a single-phase transmission line. The total cost of a three-phase line is more than 75 per cent of the cost of a single-phase line, however, since the three-

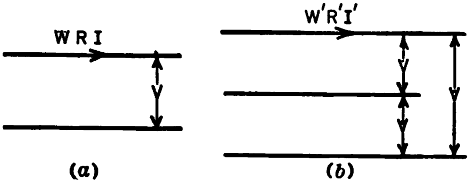


FIG. 146

phase line requires more insulators and cross-arms, and the labor cost (stringing of three wires instead of two) is greater.

The I^2R loss associated with the transmission of a given amount of power is inversely proportional to the square of the line potential and the line potential considered from this standpoint should be made as high as possible. An increase of line potential is accompanied, however, by an increased cost of insulators and transformers (described on page 128), increased spacing between wires resulting in an increased cost of construction, and by an increased corona loss (power loss due to the current conducted by the atmosphere between wires at high line potentials). Taking all factors into consideration a line potential equal to 1000 times the length of the line in miles is commonly adopted, the maximum line potential at the present time being 220,000 volts.

Transmission Line Calculations. The size of wire adopted in any case should be governed by Kelvin's Law (see Formula 34), by the relevant mechanical conditions (the tension in the wire depends upon the distance between towers, ice loading and wind pressure), and by the allowable difference in the line potentials at the generator and load ends of the line. The vector diagram of a single-phase transmission line is shown in Fig. 147, where I_l is the line current, R_l is the line resistance (both wires), X_l is the line inductive reactance (both wires), V_L is the line potential at the load, V_G is the line potential at the generator, θ_L is the phase angle between the line potential and the line current at the load, and θ_G is the phase angle between the line potential and the line current at the generator. The line resistance may be determined from a wire table and the line reactance from ωL (L is given per mile per wire by 55). The line potential at the generator from Fig. 147 is then given by

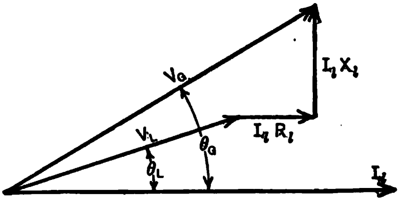


FIG. 147

133 $$V_G = \sqrt{(V_L \cos\theta_L + I_lR_l)^2 + (V_L \sin\theta_L \pm I_lX_l)^2} \text{ volts.}$$

The sign before I_lX_l is positive when the line current is in phase with or lags the line potential at the load and is negative when the line current leads the line potential at the load. It will be noted that when the line current leads the line potential at the load as in Fig. 148 the line potential at the load may be greater than the line potential at the generator. This effect is also realized in a long transmission line operated without load since the transmission line wires and the intervening atmosphere form a condenser. The capacitance of the condenser being distributed throughout the length of the line, the line current will be greatest at the generator end and zero at the load end of the line. The exact solution in this case must therefore take into account the distributed capacitance of the line.

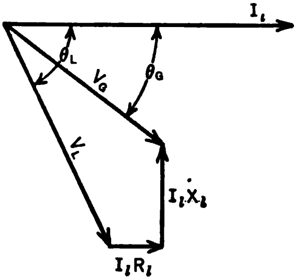


FIG. 148

The power factor ($\cos\theta_G$) at the generator end of a single-phase transmission line, from Fig. 147 or Fig. 148, is given by

134 $$\cos\theta_G = \frac{V_L\cos\theta_L + I_lR_l}{V_G}.$$

The efficiency (η) of a single-phase transmission line is given by

135 $$\eta = \frac{V_LI_l\cos\theta_L}{V_LI_l\cos\theta_L + I_l^2R_l}.$$

Three-phase potential calculations are made with the assumption that the generator and load are Y-connected as shown

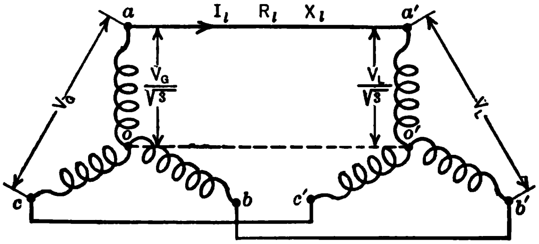


FIG. 149

in Fig. 149. Since the potential between *o* and *o'*, the respective neutral points of the generator and load, is zero by symmetry a conductor of zero resistance and reactance may be assumed to be connected between *o* and *o'*. Calculations may then be made for the single-phase circuit (*oaa'o'o*), the other two phases being disregarded. The vector diagram for this single-phase circuit may then be constructed as shown in Fig. 150, where I_l is the line current, R_l is the line resistance (one wire), X_l is the line inductive reactance (one wire), $\frac{V_L}{\sqrt{3}}$ is the Y-phase potential at the load, $\frac{V_G}{\sqrt{3}}$ is the Y-phase potential at the generator, θ_L is the phase angle between the Y-phase potential and the line current at the load and θ_G is the phase angle between the Y-phase potential and the line current at the generator. The line potential (V_G) at the generator, from Fig. 150, is then given by

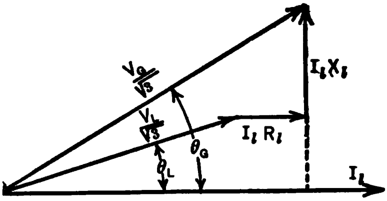


FIG. 150

136 $$V_G = \sqrt{3}\sqrt{\left(\frac{V_L}{\sqrt{3}}\cos\theta_L + I_lR_l\right)^2 + \left(\frac{V_L}{\sqrt{3}}\sin\theta_L \pm I_lX_l\right)^2}\text{ volts.}$$

The power factor ($\cos\theta_G$) at the generator end of a three-phase transmission line is given by

137 $$\cos\theta_G = \frac{\frac{V_L}{\sqrt{3}}\cos\theta_L + I_lR_l}{\frac{V_G}{\sqrt{3}}}.$$

The efficiency (η) of a three-phase transmission line is given by

138 $$\eta = \frac{\sqrt{3}\,V_LI_l\cos\theta_L}{\sqrt{3}\,V_LI_l\cos\theta_L + 3\,I_l^2R_l}.$$

CHAPTER X

THE SYNCHRONOUS GENERATOR

Construction. The synchronous generator contains an armature with a single-phase or polyphase winding and a magnetic field excited by a separate source of direct current. The armature winding may be endless (as in a direct current dynamo) but is usually open. In a single-phase generator the ends of the open winding are connected to the line and in a three-phase generator the three windings are Δ- or Y-connected (usually Y). The armature may be stationary or revolving (usually stationary for generators of more than 500 kilovolt-amperes capacity). Among the advantages of the stationary armature are better insulation, more thorough cooling (by forced ventilation), and more substantial terminal connections to the line. A revolving field may also be constructed to withstand higher centrifugal stresses than a revolving armature. The windings of a revolving armature are connected to the line through two or more slip rings and a revolving field winding is connected to its source of excitation in the same manner. The frequency of a synchronous generator equals the product of the number of poles (p) and the speed (S) in revolutions per minute divided by 120. The speed of a synchronous generator must therefore be maintained constant and is given conversely by

139 $$S = \frac{120f}{p} \text{ revolutions per minute.}$$

The maximum speed of a 60-cycle generator is then $\frac{120 \times 60}{2}$, or 3600 r.p.m., and for a 25-cycle generator $\frac{120 \times 25}{2}$, or 1500 r.p.m.

E.m.f. generated by a Synchronous Generator. The maximum e.m.f. generated in the armature of a single-phase generator when all the armature slots are filled with conductors equals the vector sum of the maximum e.m.f.'s generated in the various conductors. Since the e.m.f.'s generated in two adjacent conductors differ in phase by the subtended angle the individual e.m.f.'s may be represented by vectors of equal length drawn tangent to the surface of the armature. The vector sum of the individual e.m.f.'s is then represented to scale by the diameter of the armature (*ab* in Fig. 151). It will be noted that if the slots adjacent to *a* and *b* are left vacant the e.m.f. generated in the armature is proportional to the length (*cd*) which is nearly equal to the length (*ab*). It is then evident that the conductors in the slots adjacent to *a* and *b* should be eliminated since they contribute little to the e.m.f. of the generator and increase the armature resistance and reactance if included. It is for this reason that approximately one-third of the slots of a single-phase armature are left vacant.

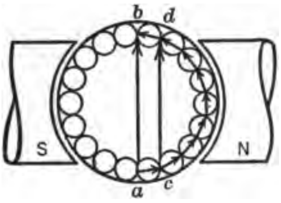


Fig. 151

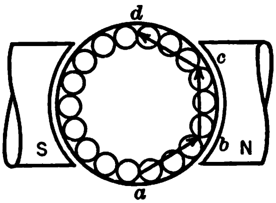


Fig. 152

In a three-phase armature the e.m.f. generated in each phase is proportional to the length (*ab*), (*bc*) and (*cd*). Since each phase belt extends over a smaller portion of the circumference of the armature all the armature slots may be filled with conductors. In a single-phase generator the entire circumference of the armature may not be used effectively and its armature must therefore be larger in diameter than the armature of a three-phase generator of the same capacity. The single-phase generator in consequence weighs approximately 30 per cent more than the three-phase generator and is therefore more expensive.

Armature Reaction in Synchronous Generators. If the alternating current established in a single turn of wire revolving at a speed of ω radians per second is in phase with the e.m.f. generated in the single turn the magnetomotive force due to the alternating current at any instant is given by $f = \mathcal{F}_m \sin \omega t$, where $\mathcal{F}_m$ is the maximum magnetomotive force due to the current. The magnetomotive force along the axis (XX') in Fig. 153 acting in conjunction with the field magnetomotive force is then given by $f = \mathcal{F}_m \sin \omega t \cos \omega t = \mathcal{F}_m \dfrac{\sin 2\,\omega t}{2}$. The magnetomotive force due to the armature current at any instant along the axis (XX') plotted with the e.m.f. generated in the rotating coil at the same instant for comparison is shown in Fig. 154. It will be noted that the frequency of the m.m.f. due to the armature current along the axis (XX') is twice that of the generated e.m.f. and successively aids and opposes the field m.m.f. The variation of the armature m.m.f. in a single-phase generator with a concentrated or distributed winding is the same as that shown in Fig. 154, the magnitude of the m.m.f. being less however in the distributed winding than in the concentrated winding. The magnetic flux produced by the armature m.m.f. is alternating and the hysteresis and eddy current loss in the armature core due to the rotation of the armature in the magnetic flux produced by the field m.m.f. is therefore augmented by the alternating flux produced by the armature m.m.f. The magnetic flux in the field poles produced by the armature m.m.f. is an alternating flux of twice the frequency of the armature current and therefore produces a large hysteresis and eddy current loss in the field poles.

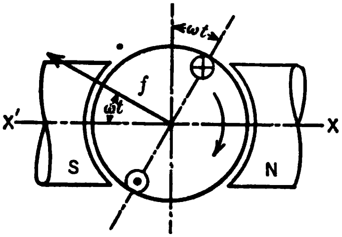


FIG. 153

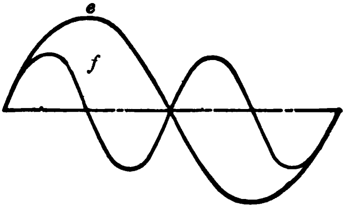


FIG. 154

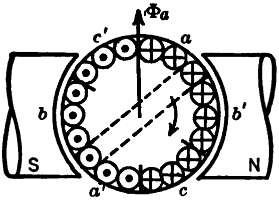


FIG. 155

If several independent single turns are wound upon an armature as shown in Fig. 155 and the e.m.f. and current in each turn is in phase the sum of the currents and their directions on each side of the interpolar plane at any instant will be constant. The magnetic flux due to these armature currents will then be constant in magnitude and direction. Since the direction of this flux (Φ_a) is perpendicular to the flux due to the field m.m.f. it will neither aid nor oppose the field flux but will distort the flux through the armature.

The conductors in a three-phase armature are connected in groups (aa', bb' and cc') as shown in Fig. 155 and will therefore produce a magnetic flux (Φ_a) which is substantially constant in magnitude and direction. The hysteresis and eddy current loss in a three-phase generator will then be much smaller than in a single-phase generator and the efficiency of a three-phase generator will therefore be higher than that of a single-phase generator.

Effect of Power Factor upon the Generated E.m.f. of a Synchronous Generator. In the single-phase generator (Fig. 153) the armature m.m.f. at any instant along the axis (XX') when the

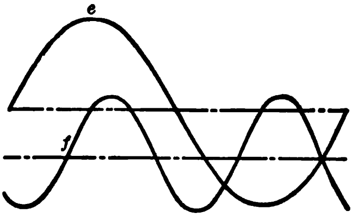


Fig. 156

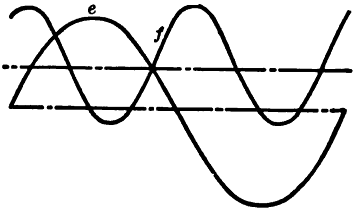


Fig. 157

armature current lags the generated e.m.f. by θ degrees is given by $f = \mathcal{F}_m \sin(\omega t - \theta) \cos \omega t = \frac{\mathcal{F}_m}{2} (\sin(2\omega t - \theta) - \sin \theta) = \frac{\mathcal{F}_m}{2} \sin(2\omega t - \theta) - \frac{\mathcal{F}_m}{2} \sin \theta$. This magnetomotive force plotted with the e.m.f. generated at the same instant is shown in Fig. 156. It will be noted that when θ equals zero (Fig. 154) the successive magnetizing and demagnetizing effects of the armature current are equal but that the demagnetizing effect with a lagging current (Fig. 156) is greater than the magnetizing effect. When the armature current leads the generated e.m.f. by θ degrees $f = \mathcal{F}_m \sin(\omega t + \theta) \cos \omega t = \frac{\mathcal{F}_m}{2} (\sin(2\omega t + \theta) + \sin \theta) = \frac{\mathcal{F}_m}{2} \sin(2\omega t + \theta) + \frac{\mathcal{F}_m}{2} \sin \theta$. This magnetomotive force plotted with the e.m.f. generated at the same instant is shown in Fig. 157. It will be noted that with a leading armature current the magnetizing effect of the armature current is greater than the demagnetizing effect. The net armature flux and the generated e.m.f., in consequence, is therefore weakened by a lagging current and strengthened by a leading current.

In a three-phase generator the plane of the flux (Φ_a) due to the armature currents will be turned in the direction of rotation through an angle (θ) when the current in each phase lags the phase e.m.f. by θ degrees, as shown in Fig. 158, and will be turned in the opposite direction through an angle (θ) when the phase current leads the phase e.m.f. by θ degrees. A lagging current then produces a demagnetizing effect and a leading current a magnetizing effect in the three-phase generator. The generated e.m.f. will therefore be decreased by a lagging current and increased by a leading current.

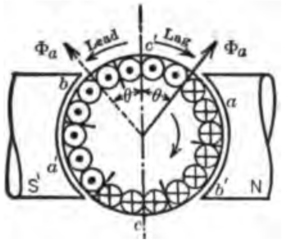


FIG. 158

Relation between Generated E.m.f. and Terminal Potential. The vector diagram for a single-phase generator is shown in Fig. 159. E_a is the generated e.m.f., V_t is the terminal potential, I_a is the armature current, θ is the phase angle between the terminal potential and the armature current, R_a is the armature resistance, and X_a is the armature inductive reactance. The generated e.m.f., from Fig. 159, is then given by

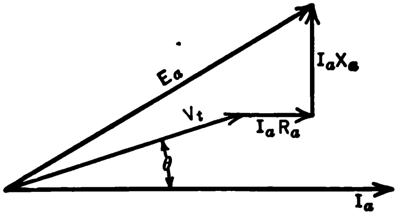


FIG. 159

$$140 \qquad E_a = \sqrt{(V_t \cos\theta + I_aR_a)^2 + (V_t \sin\theta \pm I_aX_a)^2} \text{ volts.}$$

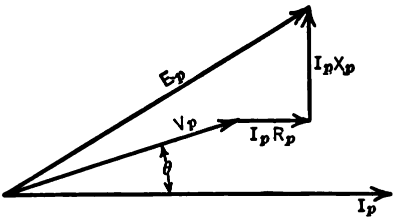


FIG. 160

The sign before I_aX_a is positive when I_a is in phase with or lags V_t and is negative when I_a leads V_t.

The vector diagram for one phase of a three-phase generator is shown in Fig. 160. E_p is the generated phase e.m.f., V_p is the phase potential, I_p is the phase current, θ_p is the phase angle between the phase potential and the phase current, R_p is the phase resistance, and X_p is the phase inductive reactance. The generated phase e.m.f., from

Fig. 160, is then given by

141 $$E_p = \sqrt{(V_p \cos\theta + I_pR_p)^2 + (V_p \sin\theta \pm I_pX_p)^2} \text{ volts.}$$

In a Δ-connected generator $E_l = E_p$, and in a Y-connected generator $E_l = \sqrt{3}\, E_p$.

Parallel Operation of Synchronous Generators. Before generator No. 2 can be connected in parallel with generator No. 1 in Fig. 161 the following conditions must be satisfied: (1) the terminal potentials, V_1 and V_2, must have the same frequency, (2) the terminal potentials must differ in phase by 180 degrees in the circuit *abcda*, and (3) the terminal potentials must be equal at any instant. If any one of these conditions is not satisfied current will flow from one generator through the other when the switch is closed. When all three conditions are satisfied the generators are said to be in synchronism. Any device connected across the open switch that will indicate the current flowing in the circuit *abcd* may be employed to synchronize the two generators.

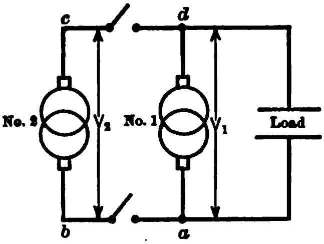


FIG. 161

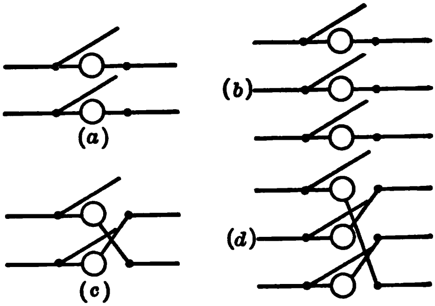


FIG. 162

In the dark lamp method incandescent lamps are connected across the open single-phase switch as shown in Fig. 162 (*a*). If the generators are not in synchronism the lamps will either burn steadily or flicker. The speed and terminal potential of each generator must then be adjusted until the lamps remain dark;

the switch may then be closed. The connections of the lamps for synchronizing three-phase generators by the dark lamp method are shown in Fig. 162 (*b*). In the bright lamp method the lamps are connected diagonally across the open switch as shown in Fig. 162 (*c*) for a single-phase switch and in Fig. 162 (*d*) for a three-phase switch. In this case synchronism is indicated when the lamps burn brightest and do not flicker. An instrument called a synchroscope, or a synchronism indicator, may be connected between the generator to be synchronized and the bus-bars. The rotary motion of a pointer on the instrument indicates whether the generator to be synchronized is running too fast or too slow. Synchronism is indicated when the pointer remains stationary in a vertical position.

After the two generators (Fig. 161) are connected in parallel the load may be distributed between the two generators by regulating the mechanical power input to each generator (admit more or less steam to a steam turbine for example) and by adjusting the generated e.m.f.'s of the two generators. The adjustment of the generated e.m.f.'s alone will effect no change in the power supplied by each generator but will change the power factor of each generator. The mechanical power input and the generated e.m.f. of each generator should be adjusted until each generator supplies its proportion of the total load and the power factor of each generator is the same and equal to the load power factor.

CHAPTER XI

THE SYNCHRONOUS MOTOR

Construction. The construction of the synchronous motor is substantially the same as that of the synchronous generator; that is, it contains a stationary or revolving single-phase or three-phase armature and a magnetic field excited by a separate source of direct current.

Starting Conditions. If an alterating current is established in an armature winding located in a magnetic field of fixed direction as shown in Fig. 163 the armature torque is alternating, clockwise for the direction of current shown in Fig. 163 and counterclockwise during the next half-cycle. There is therefore no net torque in either direction and the synchronous motor is not self-starting. If the armature (or the field, as the case may be) is rotated by an auxiliary motor at a speed of $\frac{120f}{p}$ revolutions per minute and the e.m.f. generated in the motor armature is made equal and opposite to the line potential at any instant the alternating current established in the armature winding when the line switch is closed and the auxiliary motor is disconnected will change in direction synchronously with respect to the poles so that the direction of the armature torque will remain constant. It will be noted, in Fig. 164, that the armature has turned 180 degrees from the position shown in Fig. 163 and since the current has also reversed in direction the torque is still clockwise. The torque of a single-phase

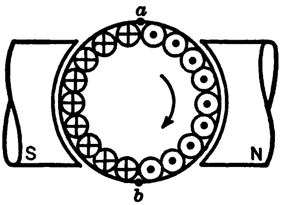


FIG. 163

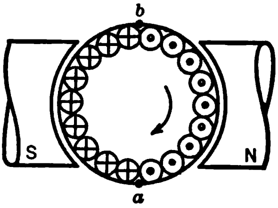


FIG. 164

motor will be pulsating but in a three-phase motor the torque will be nearly constant. In starting a synchronous motor synchronism may be indicated by the dark or bright lamp method or the synchroscope employed in synchronizing generators operated in parallel. After the synchronous motor is synchronized and connected to the line the auxiliary motor may be disconnected electrically or mechanically. The synchronous motor will then rotate under its own torque at constant speed (given by 139).

The Effect of changing the Load on a Synchronous Motor. The terminal potential (V_t) of a single-phase synchronous motor must equal the vector sum of the generated e.m.f. (E_a), the resistance potential (I_aR_a) and the inductive reactance potential (I_aX_a) as shown in Fig. 165. For the three-phase motor the same diagram may be constructed substituting V_p for V_t, E_p for E_a, I_pR_p for I_aR_a, and I_pX_p for I_aX_a. An increase of load will cause the armature to slow down momentarily and then continue to rotate at the same constant speed as before. Each conductor on the armature will then pass a given point on a pole face a little later than it did before the load was increased. The generated e.m.f., while unchanged in magnitude except through the effect of armature reaction, will then lag the terminal potential by a greater angle as shown by the dotted vector in Fig. 165. The armature current will then be readjusted in magnitude and phase until the power input to the motor equals the power output plus the losses. A momentary change in speed due to a sudden application of heavy load may cause the armature to drop out of synchronism and come to rest.

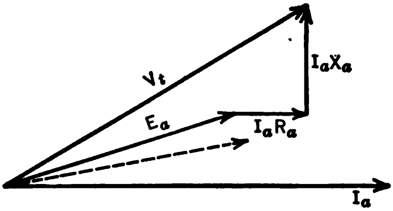


FIG. 165

The Effect of changing the Excitation of a Synchronous Motor. If the excitation of a synchronous motor operating under the conditions shown in Fig. 165 is increased the generated e.m.f. must increase and the electrical conditions will then be represented by the vector diagram shown in Fig. 166, where V_t is referred to the same axis of reference as in Fig. 165. It will be noted that an increase of the generated e.m.f. may cause the armature current to lead the terminal potential. The power factor of a synchronous motor may thus be controlled by adjustment of the field current. Since the current supplied to the motor depends upon the power factor for a given load the armature current will depend upon the field current. A plot of the armature current and the field current of a synchronous motor at full load and half load (called the V-curves of a synchronous motor) is shown in Fig. 167.

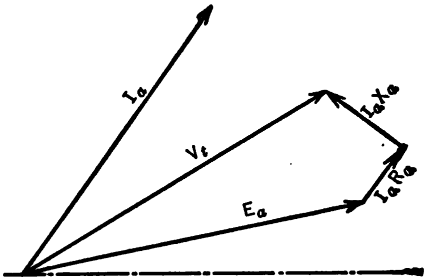


FIG. 166

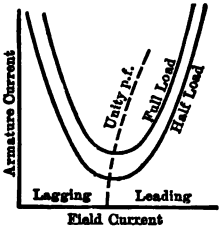


FIG. 167

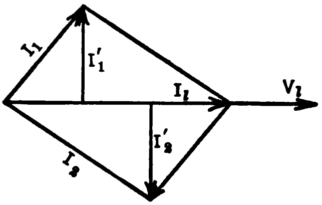


FIG. 168

When synchronous motors are operated independently the power factor of each motor may be made unity by adjustment of the field current. If one or more synchronous motors are connected in parallel with other motors which operate under a lagging power factor not subject to control, the field current of the synchronous motors may be adjusted until the wattless component (I_1') of the leading synchronous motor current equals the wattless component

(I_2') of the lagging current supplied to the other motors. The line current (I_l) in Fig. 168 will then be in phase with the line potential (V_l) (or the Y-phase potential of a three-phase line) and the system will be operated at unity power factor. A synchronous motor used for this purpose (correction of power factor) is sometimes called a synchronous condenser and may either supply mechanical power or be operated under no load. Static condensers are sometimes used in place of synchronous motors to accomplish the same effect.

CHAPTER XII

THE SYNCHRONOUS CONVERTER

Construction. A synchronous converter is similar in construction to a shunt or compound direct current generator with the addition of two or more slip rings connected to the armature winding. The synchronous converter then consists of a revolving drum wound armature with a commutator and slip rings, and a shunt or compound field winding. The single-phase converter (rarely used) has two slip rings connected to respective points in the armature winding located 180 electrical degrees apart and the three-phase converter has three slip rings connected to respective points in the armature winding located 120 electrical degrees apart.

Applications of the Synchronous Converter. In certain applications of electric power direct currents are more desirable than alternating currents; notably, in arc lighting, in electric motors requiring close speed adjustment, in electrolytic processes in which the desired chemical action depends upon the direction of the current, and in most electric railway systems due to the relevant adaptation of the series direct current motor. If a synchronous converter is synchronized and connected to an alternating current source of power it will operate at constant speed as a synchronous motor. The rotating armature will also serve as the armature of a direct current generator. Direct current can then be taken from the brushes on the commutator side, supplying power to a direct current load and also furnishing excitation to the field winding. The principal use made of the synchronous converter is in this capacity; namely, to convert alternating current power into direct current power. The same machine may be operated as a direct current shunt or compound motor and convert direct current power into alternating current power. A synchronous converter may be connected, for example, between a storage battery and an alternating current system when conditions require the maintenance of a storage battery reserve. If the armature be driven mechanically a synchronous converter may be employed to convert mechanical power into alternating and direct current power simultaneously.

Relation between the Direct E.m.f. and the Alternating E.m.f. generated in a Synchronous Converter. In a single-phase synchronous converter the e.m.f. (E_{dc}) generated between the commutator brushes equals the maximum e.m.f. (E_m) generated between the slip rings. The effective value (E_{ac}) of a sinusoidal e.m.f. generated in a single-phase converter is then given by

142 $$E_{ac} = \frac{E_{dc}}{\sqrt{2}} = 0.707\ E_{dc} \text{ volts.}$$

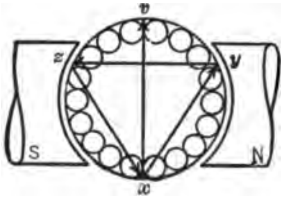


FIG. 169

If the effective value of the e.m.f. generated between two diametrically opposite points on the armature winding of a synchronous converter is represented by the vector (E_{xv}) in Fig. 169 the effective value (E_{ac}) of the phase e.m.f. generated in a three-phase synchronous converter is given by $E_{ac} = \frac{E_{xy}}{E_{xv}} \cdot \frac{E_{dc}}{\sqrt{2}}$. Since $E_{xy} = \frac{\sqrt{3}}{2} E_{xv}$, $E_{ac} = \frac{\sqrt{3}}{2} \cdot \frac{E_{dc}}{\sqrt{2}}$ and the effective e.m.f. (E_{ac}) generated between the slip rings of a three-phase synchronous converter is given by

143 $$E_{ac} = \sqrt{\frac{3}{8}}\, E_{dc} = 0.612\ E_{dc} \text{ volts.}$$

Relation between the Direct Current and the Alternating Current in a Synchronous Converter. The approximate relationship of the direct and alternating current may be determined by neglecting the copper losses and rotational losses in the converter. In a single-phase converter the alternating current power input then equals the direct current power output and, assuming unity power factor, $E_{ac}I_{ac} = E_{dc}I_{dc}$. Substituting

$$\frac{E_{dc}}{\sqrt{2}} \text{ for } E_{ac},\ \frac{E_{dc}}{\sqrt{2}} I_{ac} = E_{dc}I_{dc} \text{ and}$$

144 $$I_{ac} = \sqrt{2}\, I_{dc} = 1.41\ I_{dc} \text{ amperes.}$$

Since the maximum value (I_m) of a sinusoidal alternating current equals $\sqrt{2}$ times the effective value the maximum value of the alternating current in a single-phase converter at unity power factor is given by $I_m = \sqrt{2} \cdot \sqrt{2}\, I_{dc} = 2\, I_{dc}$.

In a three-phase converter operating at unity power factor $\sqrt{3}\, E_{ac} I_{ac} = E_{dc} I_{dc}$ or

$$\sqrt{3}\sqrt{\frac{3}{8}}\, E_{dc} I_{ac} = E_{dc} I_{dc} \text{ and } I_{ac} = \frac{1}{\sqrt{3}}\sqrt{\frac{8}{3}}\, I_{dc} \text{ or}$$

$$I_{ac} = \sqrt{\frac{8}{9}}\, I_{dc} = 0.943\, I_{dc} \text{ amperes.} \tag{145}$$

The actual value of the alternating current will be greater than the values given by 144 or 145 because the losses are not negligible. The alternating current will also be greater if the power factor is less than unity.

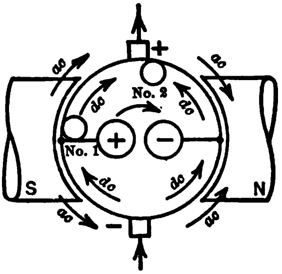


FIG. 170

The Advantages of the Polyphase Converter. In a single-phase converter (Fig. 170) the direct current enters the armature at the negative brush and flows through two paths in the armature winding to the positive brush. The alternating current (for the position shown) enters the armature from the positive slip ring and flows through two paths in the armature winding to the negative slip ring. In conductor No. 1, the alternating e.m.f. between the slip rings being zero for the position shown, the current will also be zero for operation at unity power factor and will vary during one revolution as shown in the curve (i_{ac}) in Fig. 171. The direct current flowing in conductor No. 1 will vary during one revolution as shown in curve (i_{dc}) in Fig. 171; i_{dc} being approximately equal to one-half the maximum value of i_{ac} and reversing in direction under each brush. The composite current (i_1) flowing in conductor No. 1 during one revolution is then given by the sum of i_{ac} and i_{dc} at any instant. In conductor No. 2 (located half-way between the slip ring connections) the composite current (i_2) is given at any instant by the sum of the currents i_{ac} and i_{dc} as shown in Fig. 172.

Since the heating effect of the current is proportional to i^2R at any instant it will be noted by comparing i_1 and i_2 at any instant that more power will be converted into heat in conductor No. 1 than in conductor No. 2 (actually 6.6 times as much in No. 1 in one cycle as in No. 2). The capacity of a single-phase converter is thus limited by the excessive heating of those conductors located near the slip ring connections. A similar analysis of the heating effect in the various armature conductors of a polyphase converter will show that the relative heating effect becomes more uniform as the number of phases is increased. The average heating effect in the armature winding as a whole is also decreased because the average effective value of the composite current in a polyphase converter operating at unity power factor is less than the direct current. The capacity of a synchronous converter operating at unity power factor compared with its capacity when operated as a direct current generator is given in per cent as follows:

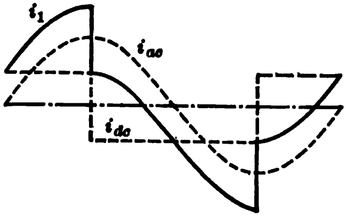


FIG. 171

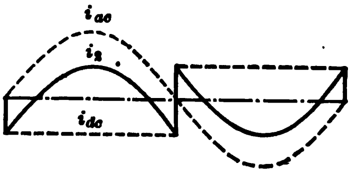


FIG. 172

No. of slip rings	2	3	4	6	12
Relative capacity	85.2	133	162	193	219.5

The six-phase converter is used extensively for this reason, the increased capacity of the twelve-phase converter being insufficient in general to compensate for its increased complexity and cost.

Motor-generators versus Synchronous Converters. The conversion of alternating current power to direct current power or the reverse may also be accomplished by coupling an alternating current motor to a direct current generator, the combination being called a motor-generator. A comparison of the synchronous converter with the motor-generator reveals certain advantages in favor of the synchronous converter; namely, higher efficiency, lower cost, and less space requirement. The motor-generator, if it be an induction motor-generator, requires no synchronizing and may therefore be started more easily than the synchronous converter. The generated e.m.f. of the generator in a motor-generator may also be varied over a wider range than the generated e.m.f. of a synchronous converter.

CHAPTER XIII

THE MERCURY ARC RECTIFIER

Construction. A highly evacuated glass chamber (Fig. 173) contains two recessed anodes (A_1A_2), a cathode (C) and an auxiliary anode (A'). The anodes (A_1A_2) are made of graphite or iron and the cathode (C) consists of a pool of mercury, a smaller pool of mercury also being located at the auxiliary anode (A'). The anodes (A_1A_2) are connected to the opposite alternating current line wires and the auxiliary anode (A') is connected to one of the line wires through a resistance. The device in which a direct current is to be established is connected betweeen the cathode and the midpoint (D) of a reactance connected between the line wires.

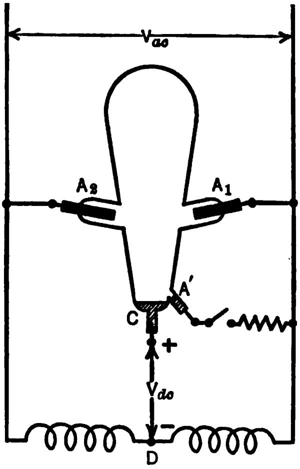


Fig. 173

Operation of the Mercury Arc Rectifier. If the tube is tipped until the mercury in the auxiliary anode (A') flows into the pool of mercury at the cathode (C) an alternating current will flow from A' to C and establish an electric arc. This arc will rise quickly to the anodes (A_1A_2) but current will flow through this arc only from each anode to the cathode and not in the reverse direction. When A_1 is at a higher potential than D, current will flow from A_1 to C to D, and when A_2 is at a higher potential than D current will flow from A_2 to C to D. A direct current will then be established in any device connected between C and D. The direct potential between C and D will equal approximately one-half the average value of each half-cycle of the alternating potential (V_{ac}) minus the potential drop in the vacuum chamber, which is about 15 volts. The direct potential (V_{dc}) is then given approximately by

146 $$V_{dc} = \frac{\sqrt{2}V_{ac}}{\pi} - 15 = 0.45\ V_{ac} - 15 \text{ volts.}$$

Applications of the Mercury Arc Rectifier. The capacity of a rectifier is limited by the temperature attained during operation. Glass tubes may be operated on 115 or 230 volt alternating current circuits at a maximum current capacity of 50 amperes; steel tubes have been made with an insulating lining which rectify up to 300 amperes. Rectifiers have been operated with a line potential as high as 15,000 volts, the efficiency of the rectifier increasing as the potential is increased. The tubes may be operated in series or in parallel when the line potential or the current to be rectified exceeds the rating of a single tube. The principal use made of the mercury arc rectifier is in connection with direct current series arc lighting systems and in charging storage batteries. It is not improbable that the rectifier will eventually displace the synchronous converter to some extent in the conversion of alternating current power to direct current power in electrolytic and railway service.

CHAPTER XIV

THE TRANSFORMER

Construction. The transformer consists of two coils of wire wound upon a magnetic core made of soft iron laminations. The core with its two windings is usually placed in a steel case filled with insulating oil, the oil also serving to conduct heat away from the windings and core by convection. The two windings, while insulated from each other, are intermingled so that their mutual inductance will be a maximum. The object of the transformer is to transform alternating current power at one potential into alternating current power at another potential. The winding which receives electric power from the source is called the primary winding and the winding which delivers electric power to the load is called the secondary winding.

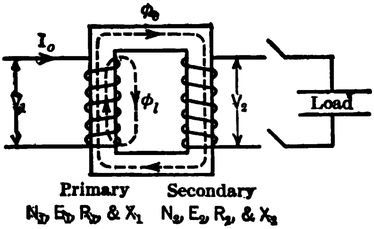


FIG. 174

Principle of Operation of the Transformer. If the primary winding of a transformer with an open secondary winding is connected to an alternating current line of V_1 volts line potential, as shown in Fig. 174, the primary current (I_0) will be small and will lag the terminal potential (V_1) nearly 90 degrees. If the resistance of the primary winding were zero and the core loss (hysteresis and eddy current losses in the iron core) were zero the terminal potential (V_1), primary current (I_0) and induced e.m.f. (E_1') in the primary winding would be represented by the vector diagram shown in Fig. 175. In this case the primary current actually lags the terminal potential by 90 degrees and is of low magnitude $\left(I_0 = \dfrac{V_1}{L\omega}\right)$. If the resistance of the winding and the core loss are considered the primary current will be sensibly of the same order of magnitude but will lag the terminal potential less than 90 degrees as shown in Fig. 176.

The alternating magnetic flux which induces the e.m.f. (E_1') in Fig. 175 may be resolved into two components (Fig. 174); the leakage flux (ϕ_l) which circulates through the primary winding and the surrounding air, and the core flux (ϕ_c) which circulates through the core only and therefore links both the primary and secondary windings. If the self-inductance (L_1) of the primary winding is determined by $\frac{N_1\phi_l}{i_0 \times 10^8}$ the reactance potential in the primary winding will be given by $I_0 2\pi f L_1$ or I_0X_1. The impressed primary potential (V_1) at no load must then equal the vector sum of the constituent potentials, I_0R_1, I_0X_1 and E_1, as shown in Fig. 176 where E_1 equals the effective value of $\frac{N_1}{10^8}\frac{d\phi_c}{dt}$.

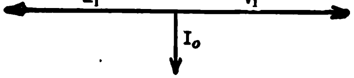


FIG. 175

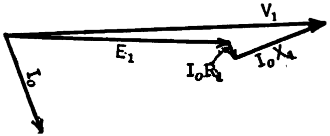


FIG. 176

Since the alternating core flux links both primary and secondary windings the electromotive force induced in each winding per turn will be the same. The ratio of the primary e.m.f. to the secondary e.m.f. will then be given by

147 $$\frac{E_1}{E_2} = \frac{N_1}{N_2} = T_1,$$

where T_1, the ratio of the number of primary to secondary turns, is called the ratio of transformation (primary to secondary).

If a load is now connected to the secondary winding the secondary current will weaken the core flux (ϕ_c), reduce the primary e.m.f. (E_1) and increase the primary current. Equilibrium will be established when the primary no load current (I_0) is augmented by an additional primary current (I_1') of such magnitude that $N_1I_1' = N_2I_2$. The demagnetizing effect of the secondary ampere turns (N_2I_2) will then be counterbalanced by the magnetizing effect of a component of the primary ampere turns (N_1I_1'). The complete vector diagram of the loaded transformer is then represented by Fig. 177. The primary current (I_1), the vector sum of I_0 and I_1', is only slightly greater than I_1'. Then $N_2I_2 = N_1I_1$ (approximately) and

148 $$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{T_1} \text{ (approximately).}$$

The secondary terminal potential (V_2) equals the secondary e.m.f. (E_2) minus (vectorially) the secondary reactance potential (I_2X_2) due to the alternating secondary leakage flux, and the secondary resistance potential (I_2R_2). The power factor of the transformer at the primary terminals equals $\cos \theta_1$ and the power factor of the secondary load equals $\cos \theta_2$.

Transformer Calculations. The relationship of the primary and secondary terminal potentials of a loaded transformer may be obtained from Fig. 177 but the calculation may be simplified by substituting composite or equivalent values of resistance and reactance for the individual resistances and reactances of the two windings. The composite resistance and composite reactance of a transformer may be determined experimentally as follows. The power input (P_1) and the primary current (I_1) of a transformer with the secondary short-circuited is measured as shown in Fig. 178 with the primary potential (V_1) reduced to such a magnitude that the primary current (I_1) does not exceed the normal rating of the primary winding. Since the core loss of a transformer operating under reduced primary potential is small the power input (P_1) equals the copper losses (approximately) in the two windings. The composite resistance (R_{c1}) of the transformer from the primary side is then given by

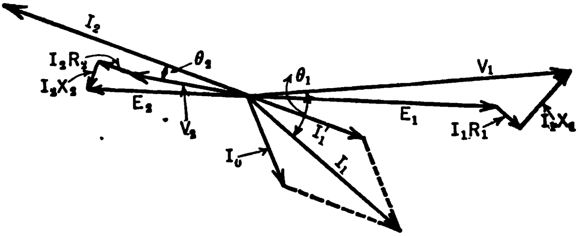


FIG. 177

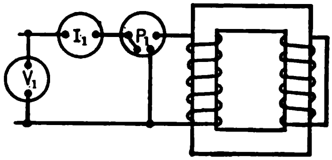


FIG. 178

149 $$R_{c1} = \frac{P_1}{I_1^2} \text{ ohms.}$$

The composite impedance (Z_{c1}) of the transformer from the primary side is given by

150 $$Z_{c1} = \frac{V_1}{I_1} \text{ ohms}$$

and the composite reactance (X_{c1}) of the transformer from the primary side is given by

151 $$X_{c1} = \sqrt{(Z_{c1})^2 - (R_{c1})^2} \text{ ohms.}$$

Since $I_1^2R_{c1} = I_2^2R_{c2}$ where R_{c2} is the composite resistance of the transformer from the secondary side and since $I_1^2 = \frac{I_2^2}{T_1^2}$ (approximately), then

152 $$R_{c2} = \frac{R_{c1}}{T_1^2} \text{ ohms.}$$

The composite reactance of the transformer from the secondary side is given by

153 $$X_{c2} = \frac{X_{c1}}{T_1^2} \text{ ohms.}$$

An equivalent transformer may then be substituted for the actual transformer as shown in Fig. 179. The equivalent transformer possesses no primary resistance or reactance, the secondary resistance is R_{c2} and the secondary reactance is X_{c2}. The vector diagram for this equivalent transformer is shown in Fig. 180. Then for any value of V_2, I_2 and θ_2, V_1 is given by

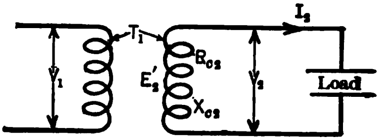


FIG. 179

154 $$V_1 = T_1\sqrt{(V_2 \cos\theta_2 + I_2R_{c2})^2 + (V_2 \sin\theta_2 \pm I_2X_{c2})^2} \text{ volts.}$$

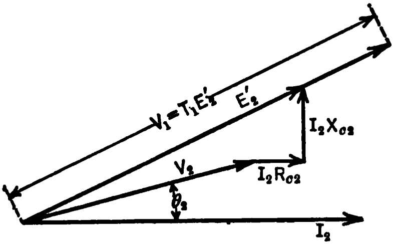


FIG. 180

If the power input to a transformer is measured as shown in Fig. 178 but at rated primary potential with the secondary open-circuited the core loss will be given approximately by P_1 since the copper loss at no load is small compared with the core loss. The core loss (P_c) may thus be determined experimentally and the efficiency of the transformer under any condition of secondary load is given by

155 $$\eta = \frac{V_2 I_2 \cos \theta_2}{V_2 I_2 \cos \theta_2 + I_2^2 R_{e2} + P_c}.$$

Special Types of Transformer. A transformer with a fixed primary winding and a movable counter-weighted secondary winding, called a constant current transformer, is shown in Fig. 181. When the secondary current is zero the secondary winding rests upon the primary winding and is linked by most of the flux produced by the primary current. If a load is connected to the secondary winding a force of repulsion between the primary and secondary currents causes the secondary winding to rise. The linkage flux and secondary e.m.f. are then reduced in consequence and the secondary current will not increase as much as it would if the secondary winding were fixed with respect to the primary winding. A change in the resistance of the secondary circuit is thus accompanied by a corresponding change in the secondary e.m.f. due to a change in the position of the secondary winding. The secondary current will then remain sensibly constant for a moderate change in the secondary resistance. Constant current transformers are used extensively in connection with series street lighting systems.

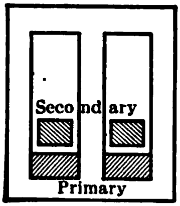


FIG. 181

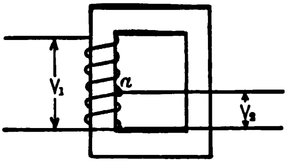


FIG. 182

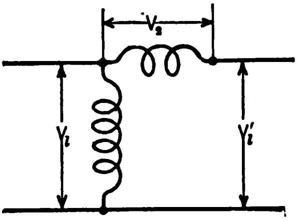


FIG. 183

A transformer in which part of the primary winding serves as the secondary winding (or the reverse), called an auto-transformer, is shown in Fig. 182. The principal advantage of this type of transformer is in the saving of a part of the weight of copper wire required in the ordinary transformer. If an adjustable contact is provided at a the secondary potential (V_2) may be varied from zero to V_1. This arrangement is frequently used in a device called a compensator employed in starting induction motors. The induction regulator used for the adjustment of line potential in transmission lines consists of an auto-transformer connected as shown in Fig. 183. The secondary winding in this case is mounted upon a shaft so that the secondary winding may be turned with respect to a fixed primary winding. The linkage flux may then be varied from zero to a maximum and the line potential (V_l') may be changed gradually from V_l to $V_l \pm V_2$.

Transformer Connections in Three-phase Circuits. A three-phase transformer may be constructed, as shown in Fig. 184 for example, containing three primary windings and three secondary windings connected respectively in Y or Δ. The cost of such a transformer is less than that of three single-phase transformers serving the same purpose and it has a higher efficiency. Single-phase transformers connected in Y or Δ are used more extensively than three-phase transformers, however, because single-phase reserve units are less expensive, high temperature in one single-phase transformer will not injure the other transformers and any single-phase transformer may be repaired while the other transformers remain in service. When two sets of transformers are connected in parallel to the primary and secondary circuits of a three-phase system any combination of Δ or Y connection may be employed in each set except that with one set of transformers connected ΔY or YΔ the other set may not be connected ΔΔ or YY.

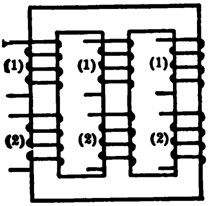


Fig. 184

CHAPTER XV

THE THREE-PHASE INDUCTION MOTOR

The Production of a Rotating Magnetic Flux. The magnetic flux traversing the space between the two rotating magnetic poles, shown in Fig. 185, may be represented by a vector of constant length rotating at the same speed as the rotating poles. If the axes of two stationary coils of wire are displaced by 90 degrees and the sinusoidal currents (i_1 and i_2) established in the two coils differ in phase by 90 degrees the magnetic flux along the axis XX' in Fig. 186 may be represented by $\phi_{xx'} = \Phi_m \sin \omega t$ and along the axis YY' by $\phi_{yy'} = \Phi_m \sin (\omega t + 90°) = \Phi_m \cos \omega t$, the maximum flux produced by each coil being assumed the same. The resultant magnetic flux (ϕ_r) through O, the intersection of the two axes, at any instant is then given by

$$\phi_r = \sqrt{(\Phi_m \sin \omega t)^2 + (\Phi_m \cos \omega t)^2}$$
$$= \Phi_m \sqrt{\sin^2 \omega t + \cos^2 \omega t} = \Phi_m.$$

The angle (α) between the direction of the flux and the YY' axis at any instant is given by $\alpha = \tan^{-1} \dfrac{\sin \omega t}{\cos \omega t} = \tan^{-1} \tan \omega t = \omega t$. The magnetic flux through O will then be of constant magnitude (Φ_m) and will rotate at a constant angular velocity (ω). It will be noted that a rotating magnetic flux similar to that shown in Fig. 185 may be produced by two or more stationary coils of wire conducting sinusoidal currents of equal magnitude and frequency differing in phase respectively by the angle of displacement between the axes of the coils.

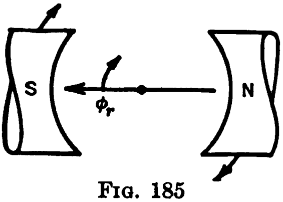


FIG. 185

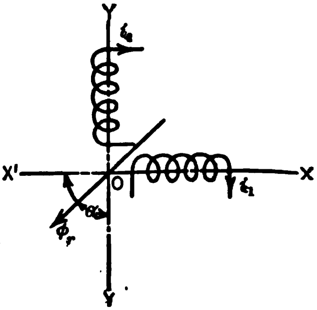


FIG. 186

If three windings located 120 degrees apart are placed in the slots of a magnetic core as shown in Fig. 187 and the currents established in the three windings differ in phase by 120 degrees a rotating flux of constant magnitude and constant angular velocity will be produced. When the current in the phase *aa′* is a positive maximum the position of the rotating flux is represented by vector (1) and by vectors (2) and (3) when the currents in phases *bb′* and *cc′* respectively are a positive maximum. In Fig. 187 there is one complete set of three-phase windings and the rotating flux makes one revolution in the cyclical period (*T* seconds) of the three-phase currents. When there are two sets of three-phase windings as shown in Fig. 188 the rotating flux moves from the position of vector (1) to vector (4), for example, during a cyclical period and makes a complete revolution in 2 *T* seconds. The angular velocity (S_f) of the rotating flux then depends upon the frequency (f) of the three-phase currents and the number of phase sets (m), and is given by

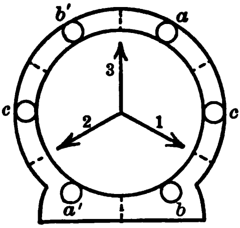


FIG. 187

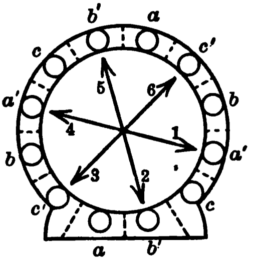


FIG. 188

156 $$S_f = \frac{60\,f}{m} \text{ revolutions per minute.}$$

Principle of Operation of the Three-phase Induction Motor. If three windings are placed in the slots of a laminated iron core as shown in Figs. 187 or 188 and are connected in Y or Δ to a three-phase source of power a rotating flux of angular velocity (S_f), given by 156, will be produced within the space surrounded by the three windings. The direction of the e.m.f. generated in any conductor located within this space will be downward at the instant shown in Fig. 189 and the direction of the current established in the conductor by the generated e.m.f. when the ends

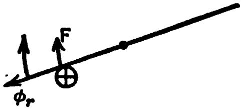


FIG. 189

of the conductor are connected will also be downward. The electromagnetic force (F) acting on such a conductor will then be in the same direction as the rotating flux and the conductor, if free to move, will follow the rotating flux. Each conductor on the rotating member of a three-phase induction motor is thus acted upon by a force which causes the rotating member to turn in the same direction as the rotating flux.

Construction of the Three-phase Induction Motor. The primary structure, called the stator, contains a three-phase Y- or Δ-connected winding embedded in the slots of a laminated iron core and is similar to the stationary armature of a three-phase synchronous generator. The secondary structure, called the rotor, contains either a bar or a wire winding embedded in the slots of a laminated iron core which is keyed to the shaft or a spider attached to the shaft. In the bar winding rotor, called a squirrel cage rotor, copper bars embedded in the slots are connected at each end of the rotor by copper rings. In the wire winding rotor, called a wound rotor, three groups of insulated copper wires are embedded in the rotor slots and are Y-connected. The terminals of the three windings of the wound rotor are either short-circuited or connected through three slip rings to an external Y-connected rheostat. The stator may contain one or more phase sets depending upon the desired rotor speed. It will be noted that the rotor windings are not connected in any way to the three-phase source of power.

Relation between Load and Speed of the Rotor. At no load the rotor torque must only be sufficient to counterbalance the reactive torque due to the rotational losses and is therefore small. The rotor will then turn at such a speed at no load that the e.m.f. generated in the rotor winding will establish a rotor current of sufficient magnitude to produce the required no load torque. Since the generated e.m.f. in the rotor depends upon the relative speed of the rotor and the rotating flux and the e.m.f. required at no load is small the rotor speed at no load will nearly equal the flux speed. It will be noted that the rotor speed cannot actually equal the flux speed at any time since there would then be no relative motion and the rotor e.m.f. would be zero.

An increase of load (or reactive torque) will cause the rotor to slow down until the increased rotor e.m.f. establishes sufficient current to produce the required electromagnetic torque. The speed of an induction motor is usually indicated by the ratio of the difference between the flux speed (S_f) and the rotor speed (S_r), and the flux speed (S_f). This ratio, called the slip (s) and expressed in per cent, is given by

157 $$s = \left(1 - \frac{S_r}{S_f}\right) 100 \text{ per cent.}$$

At no load the slip is a fraction of a per cent and at full load the slip ranges from 5 to 15 per cent, decreasing as the rating of the motor increases. In very large motors the slip is sometimes less than 5 per cent at full load.

At no load the stator current must be of sufficient magnitude to produce the rotating flux and provide the power absorbed by the no load losses. The no load stator current in its magnetizing action resembles the no load primary current of a transformer in that it lags the terminal potential to a considerable degree but differs in respect to magnitude, the ratio of no load stator current to full load stator current in the induction motor being much greater than in the transformer. This is due to the high reluctance of the air gap traversed by the rotating flux in the motor. To reduce this reluctance and incidentally the no load stator current, induction motors are usually constructed with a very small clearance between the rotor and the stator, often only a few hundredths of an inch.

The induction motor for the reasons stated operates at no load with a comparatively large stator current and at a very low power factor. The rotor reaction in an induction motor, corresponding to the armature reaction in the machines discussed previously, is demagnetizing and its effect is similar to that of the secondary ampere turns of a transformer. An increase in load therefore weakens the rotating flux, reduces the stator e.m.f. generated by the rotating flux and increases the stator current. Since the component of the stator current which balances the demagnetizing rotor current is nearly in phase with the stator potential an increase in load is accompanied by an increase in power factor. The improvement in power factor with an increase in load is restricted, however, by the abnormal magnitude of the stator magnetizing current and the power factor at full load is seldom greater than

90 per cent. The typical operating characteristics of a three-phase induction motor are shown in Fig. 190.

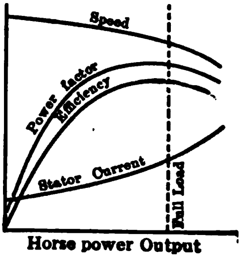


FIG. 190.

Starting Conditions in the Three-phase Induction Motor. With the rotor at rest the induction motor resembles a short-circuited transformer. The phase angle (θ) between the rotor e.m.f. and the rotor current is given by $\tan \theta = \frac{2\pi f L}{R}$. The frequency ($f$) of the rotor e.m.f. at standstill equals the frequency of the stator terminal potential and since the resistance of the rotor (R) is small compared with the rotor reactance ($2\pi f L$) the rotor current will lag considerably behind the rotor e.m.f. at standstill. The rotor current in any rotor conductor will not reach its maximum value then until the denser part of the rotating flux has passed on. The starting torque of the squirrel cage induction motor with full load current in the stator winding will therefore be only a small fraction of the rated full load torque. In the wound rotor induction motor, equipped with rotor slip ring connections and an external rheostat, resistance (R_e) may be added to each rotor winding so that $\tan \theta \left(= \frac{2\pi f L}{R + R_e} \right)$ may be made smaller during the starting period and the starting torque increased. Since an increase of rotor resistance also reduces the rotor current and in this way reduces the rotor torque only sufficient resistance should be added to the rotor circuit to obtain a maximum torque.

As the rotor comes up to speed the external resistance may be reduced to zero since the frequency (sf) of the rotor e.m.f. decreases as the speed increases.

The addition of external resistance to the rotor windings of an induction motor during the starting period serves also to reduce the stator current since the rotor is then not short-circuited. Excessive starting current in a squirrel cage induction motor must be prevented by connecting resistances or a three-phase auto-transformer with adjustable contacts, called a compensator, between the stator terminals and the three-phase source of power. In some cases induction motors are started without excessive current by connecting the stator windings first in Y and then in Δ.

In starting a synchronous motor as an induction motor the field winding must first be opened in several places so that the e.m.f. generated between any two points will not be excessive. Short-circuited copper bars built into the pole faces take the place of a rotor winding. Three-phase currents established in the armature of the synchronous motor (operated at reduced terminal potential) will then produce a rotating flux and cause either the armature or the field to revolve in accordance with the induction motor principle. When the rotating member reaches its maximum speed (nearly synchronous speed) the direct current field circuit is closed and the terminal potential is increased to full line potential. The rotating member will then be pulled into synchronism and the synchronous motor will operate continuously at constant speed.

Speed Adjustment of the Three-phase Induction Motor. The inherent speed variation of the induction motor with changes in load (see Fig. 190) resembles that of the direct current shunt motor; that is, the speed decreases slightly as the load is increased. The speed may be adjusted slightly by changing the magnitude of the terminal potential; a reduction of the terminal potential weakens the rotating flux causing the rotor to revolve at decreased speed for the same electromagnetic torque. In the wound rotor type of induction motor the speed may be adjusted by changing the resistance of the rotor circuit by means of an external rheostat connected to the rotor windings by slip rings; an increase of the rotor resistance reduces the speed of the rotor. Abrupt changes in speed may be obtained by changing the number of phase sets but this method involves increased expense in the construction of the motor and complicated connections. The speed of two induction motors with their rotors coupled mechanically may be changed as a unit by connecting the stator winding of one motor to the rotor winding of the other motor through slip rings. The motors are then said to be connected in concatenation or cascade. If the number of phase sets in each motor is represented by m_1 and m_2 respectively, four speeds may be obtained as follows: $S_r = \frac{60f}{m_1}(1 - s)$ (motor No. 1 operated alone), $S_r = \frac{60f}{m_2}(1 - s)$

(motor No. 2 operated alone), $S_r = \frac{60f(1-s)}{m_1 + m_2}$ (motors connected in opposition in concatenation), and $S_r = \frac{60f\ (1-s)}{m_1 - m_2}$ (motors connected in conjunction in concatenation). The speed of an induction motor may also be adjusted by changing the frequency of the terminal potential but this is rarely possible.

The Three-phase Induction Generator. If the rotor conductors of a three-phase induction motor are driven mechanically at a higher speed than the rotating flux the rotor e.m.f. and current are reversed and the rotor magnetomotive force aids the stator magnetomotive force. The stator e.m.f. may then exceed the stator terminal potential and electric power will be delivered to the line. Since the operation of the machine as an induction generator is dependent upon the production of a rotating flux by an external three-phase source of power the induction generator may not be operated alone but must be connected in parallel with a synchronous generator. The principal advantage of the induction generator relates to its ability to withstand a short circuit at its terminals without serious injury. A short circuit eliminates the source of the rotating flux and therefore causes generator action to cease immediately. In some instances induction motors on electric locomotives are driven above synchronous speed when going down grade and furnish regenerative braking.

The Single-phase Induction Motor. If a single-phase current is established in the stator winding of a single-phase induction motor the net torque developed by the stationary rotor at any instant, as shown in Fig. 191, will be zero. If the rotor be turned mechanically the displacement of the rotor will cause the rotor to develop torque in the direction of displacement as shown in Fig.

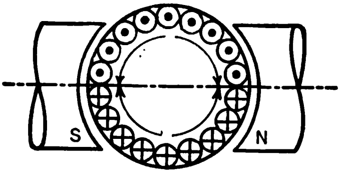


FIG. 191

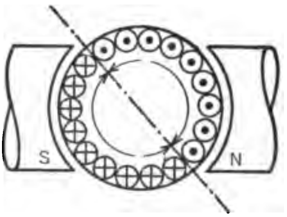


FIG. 192

192. The rotor will then come up to speed and an additional e.m.f. will be generated in the rotor conductors due to their motion in the alternating magnetic flux. The current established by the composite effect of the induced e.m.f. and the rotational e.m.f. will cause the torque to increase with the speed until a certain speed is reached where a further increase in speed causes the torque to decrease. The speed at which the torque is a maximum depends upon the ratio of the rotor reactance (at rest) to the rotor resistance; the higher the ratio of rotor reactance to rotor resistance, the higher the speed will be at which the torque is a maximum. The torque is zero for any ratio of rotor reactance to rotor resistance at a speed slightly less than synchronous speed and when the ratio of rotor reactance to rotor resistance is not greater than unity no motor torque is obtained at any speed. After being started the single-phase induction motor will operate in substantially the same manner as the three-phase induction motor. It may in fact be shown that the composite effect of the stator and rotor currents (with the rotor in motion) is to produce a rotating flux and the operation of the motor may be explained on this basis.

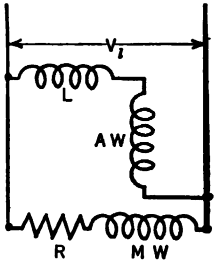


FIG. 193

The single-phase induction motor may be made self-starting by means of a split-phase stator winding. An auxiliary winding (*AW*), in Fig. 193, is placed upon the stator core with its axis displaced from the axis of the main winding (*MW*) by 90 degrees. During the starting period both windings are connected across the line, the auxiliary winding in series with inductance and the main winding in series with resistance. The currents established in the two windings will then differ in phase by nearly 90 degrees and a rotating flux will be produced as described on page 134. After the rotor comes up to speed the auxiliary winding is disconnected and the main winding connected across the line.

CHAPTER XVI

THE ALTERNATING CURRENT SERIES COMMUTATOR MOTOR

Limitations in the Use of Synchronous and Induction Motors. The synchronous motor is not adjustable as to speed, the single-phase motor is not self-starting and the three-phase motor may only be started as an induction motor at low torque. A direct and an alternating current source of power is required for synchronous motors (the direct current generator may be driven by the motor) and synchronous motors must be synchronized with the line potential if started by means of an auxiliary motor. The speed of an induction motor may be adjusted to some extent but the adjustment feature usually involves increased cost and decreased efficiency and rating. Full load starting torque may only be obtained without excessive stator current with a wound rotor three-phase induction motor with adjustable rotor resistance. It is often desirable that an alternating current motor be employed which will be adjustable as to speed and will furnish heavy starting torque without excessive current. In many instances single-phase power only is available and when an electric railway is supplied from an alternating current source of power single-phase electrification is preferred because three-phase connections by trolley wire or third rail are more complicated and expensive. In motor-driven devices intended for attachment to house circuits it is desirable that the motor be adapted for operation with either direct or alternating current.

The Alternating Current Series Commutator Motor. A direct current series motor will operate on an alternating current system since the armature current and the field flux reverse simultaneously giving unidirectional torque. The operation of an unmodified direct current series motor with an alternating current will be accompanied, however, by excessive heating of the field core, low power factor and destructive commutation. The series commutator motor must therefore be redesigned for use on alternating current circuits. The heating of the field core is reduced (1) by laminating the field core with iron sheets of low hysteresis loss, (2) by increasing the cross-sectional area of the field core and operating the motor at low flux density and (3) by operating the motor when possible on a low frequency circuit.

The vector diagram of the alternating current series motor is shown in Fig. 194. The constituent parts of the terminal potential (V_t) required to counterbalance the reactive potentials in the motor circuit are E_a (the generated e.m.f.), I_aR_a (the armature resistance potential), I_aR_s (the series field resistance potential), I_aX_a (the armature reactance potential) and I_aX_s (the series field reactance potential). Improvement of power factor in the series motor is obtained (1) by making the number of series field turns as small as possible (reduces X_s), (2) by placing a series compensating winding on the field frame so located that its magnetomotive force opposes the armature magnetomotive force (reduces I_aX_a) and (3) by operating the motor when possible on a low frequency circuit (reduces X_a and X_s).

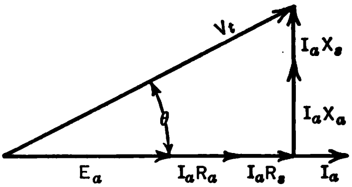


FIG. 194

In the alternating current series motor it is impossible to make the potential between *a* and *b* (Fig. 55, page 50) zero at the instant the brush breaks contact with segment No. 1 since the armature conductors connected to segments No. 1 and No. 2 carry an alternating current which is increased by the e.m.f. induced by the alternating flux when the two segments are bridged by the brush. Since the current broken when the brush breaks contact with segment No. 1 cannot be made zero improvement of commutation may only be obtained by making the current flowing from *a* to *b* through segment No. 1 as small as possible. This is accomplished (1) by reducing the alternating field flux and, consequently, the e.m.f. induced in the armature conductors short-circuited by the brush, (2) by reducing the number of armature conductors connected between the commutator segments, (3) by introducing resistance in the connections between the commutator segments and the armature conductors and (4) by operating the motor when possible on a low frequency circuit.

The speed of an alternating current series motor may be adjusted by connecting resistance in series with the motor or by connecting the motor terminals to the secondary of a transformer with adjustable secondary contacts. It will be noted that the modified series motor will operate on either an alternating current or a direct current circuit and for this reason is sometimes called the universal motor. Some alternating current single-phase motors are started as series motors and after coming up to speed a centrifugal device disconnects the field winding from the armature circuit and short-circuits the armature. The motor will then operate as a single-phase induction motor at sensibly constant speed.

CHAPTER XVII

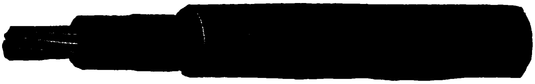
ILLUSTRATIONS OF ELECTRICAL APPARATUS

Rubber Insulation covered with Weatherproof Braid.

Weatherproof Insulation.

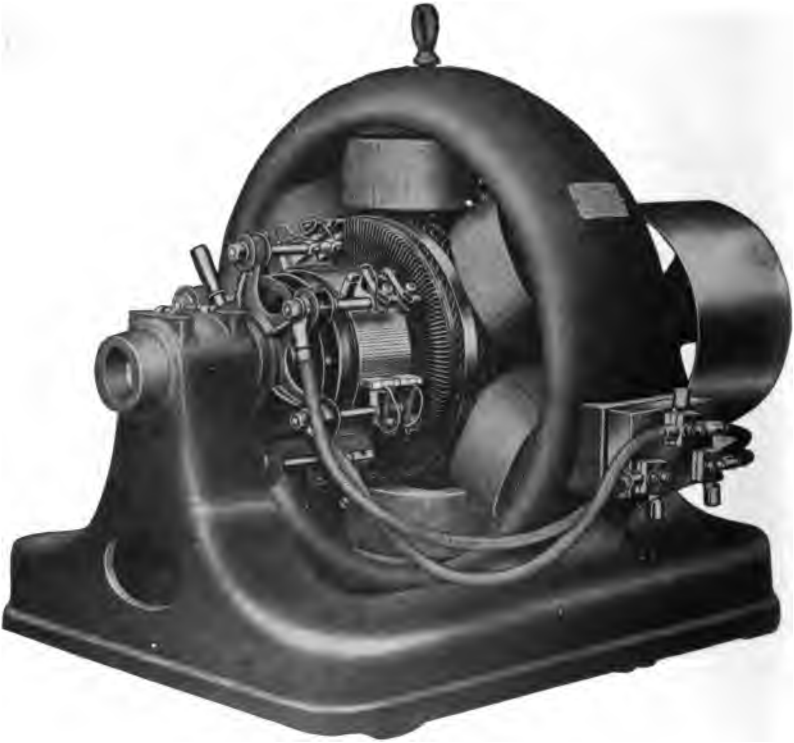
Slow-Burning Insulation.

Varnished Cambric Insulation covered with Weatherproof Braid.

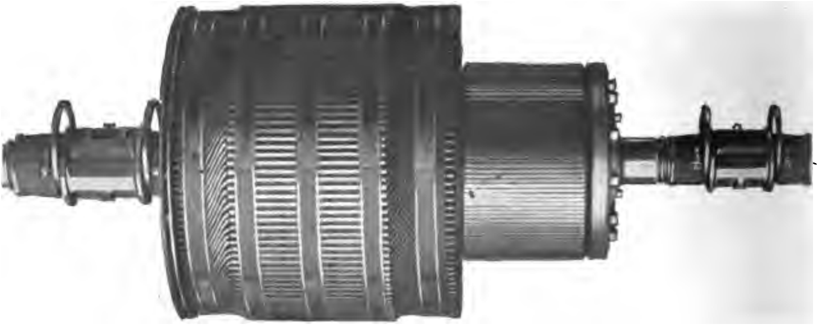


Varnished Cambric Insulation covered with a Lead Sheath.

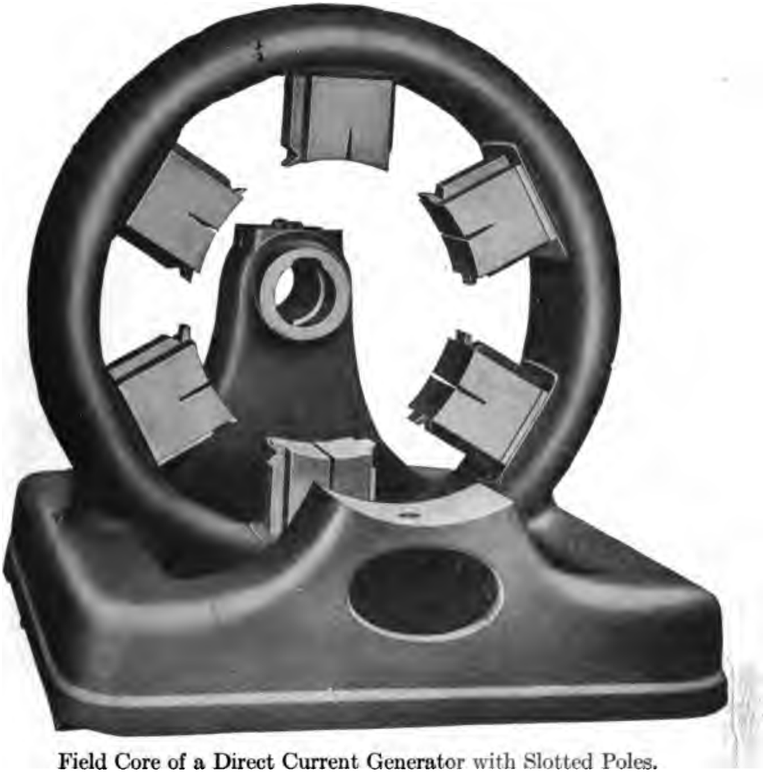
Impregnated Paper Insulation covered with a Lead Sheath.



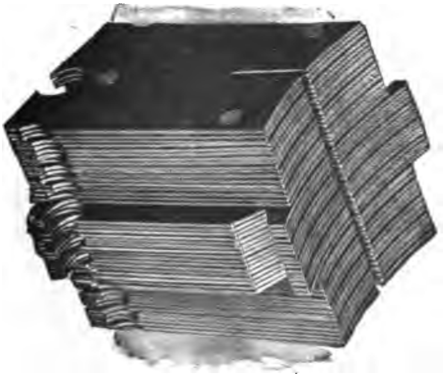
Six-Pole Direct Current Generator.



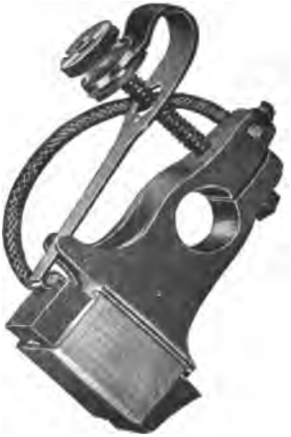
Armature of a Direct Current Generator.



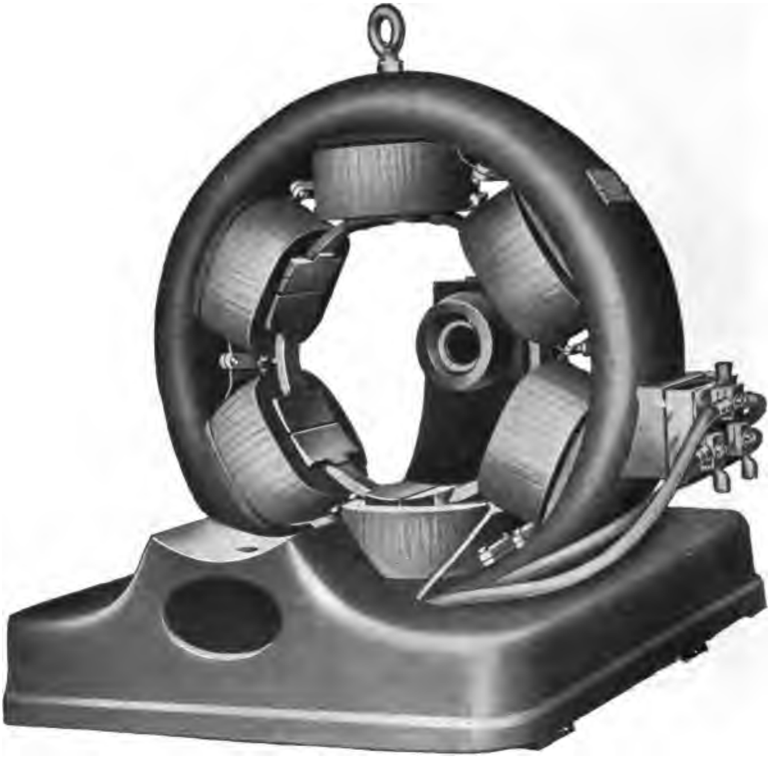
Field Core of a Direct Current Generator with Slotted Poles.



Laminated Pole Piece before being Cast-Welded into the Frame.

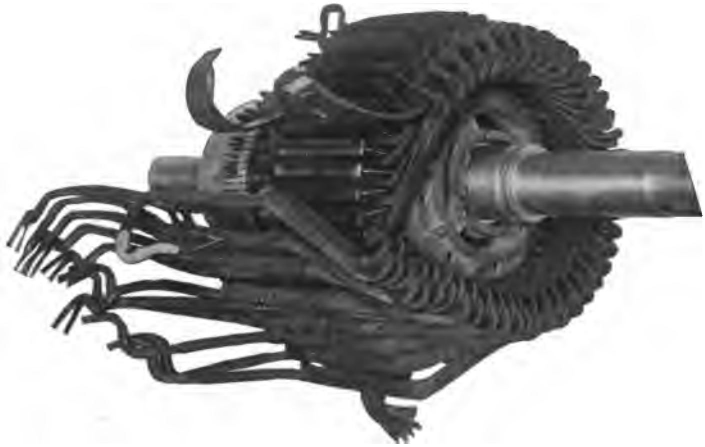


Single Brush Holder.

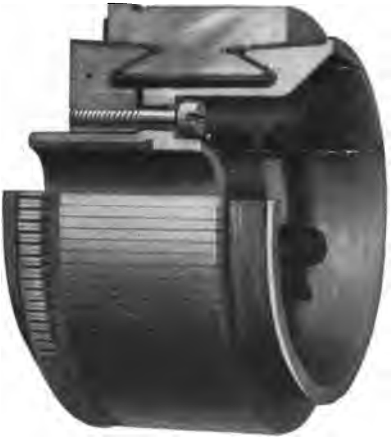


Field Assembly of a Six-Pole Direct Current Generator with Slotted Poles.

End Shields and Field Core of a Small Direct Current Shunt Motor.

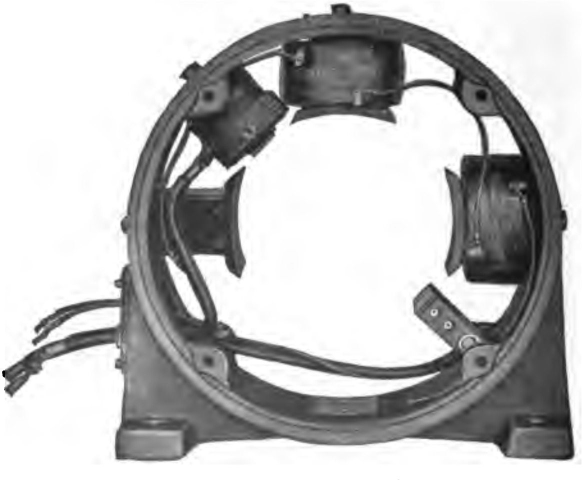


Partly Wound Armature showing Method of Assembling Coils.

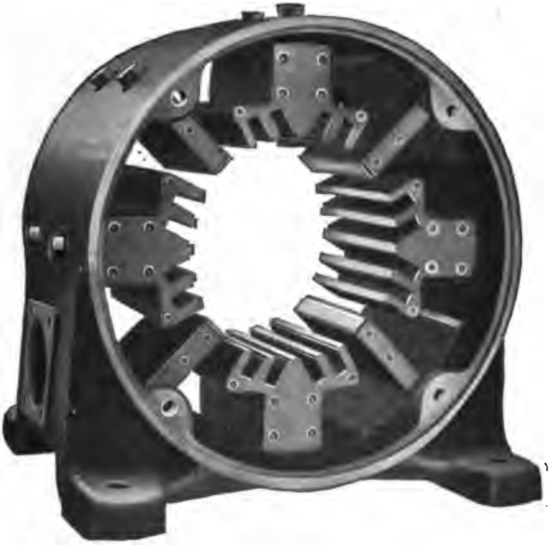


Details of Commutator Construction.

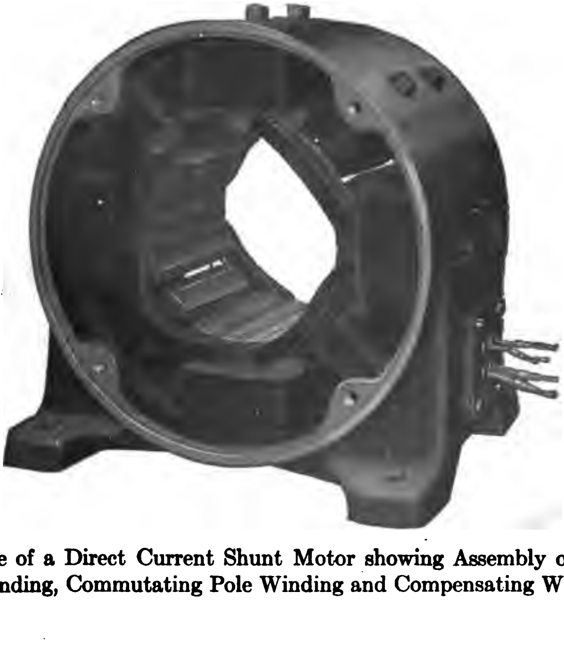
Direct Current Shunt Motor.



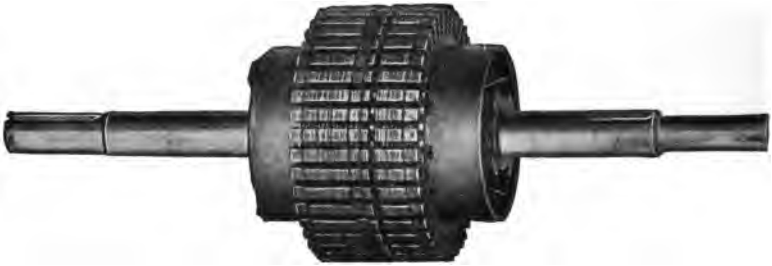
Partly Assembled Field Winding of a Direct Current Shunt Motor with Commutating Poles.



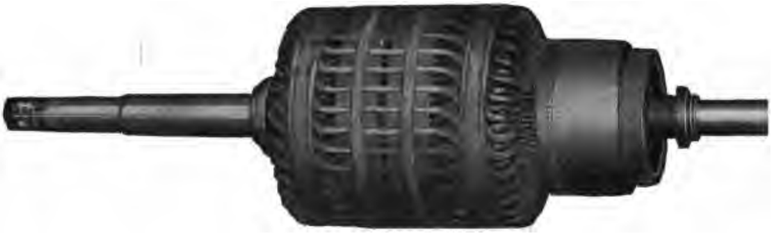
Field Core of a Four-Pole Direct Current Motor showing Commutating Poles and Slots for a Compensating Winding.



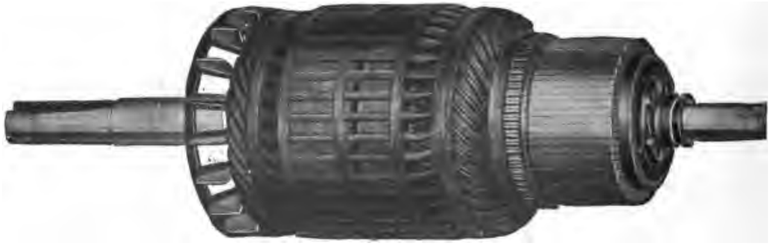
Field Core of a Direct Current Shunt Motor showing Assembly of Shunt Field Winding, Commutating Pole Winding and Compensating Winding.



Armature Core.



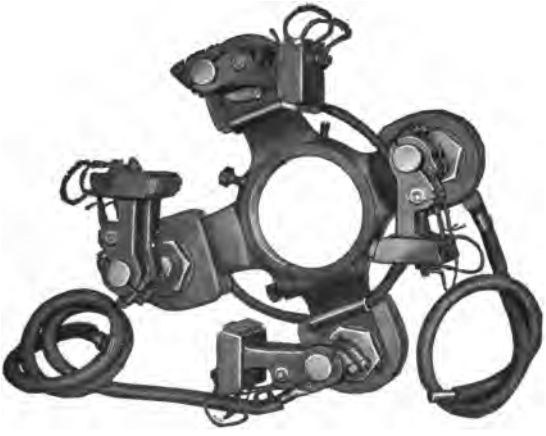
Wound Armature.



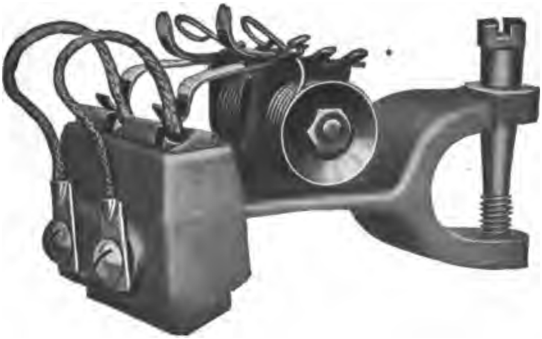
Wound Armature with Ventilating Fan.

Main Field Coil of a Direct Current Shunt Motor.

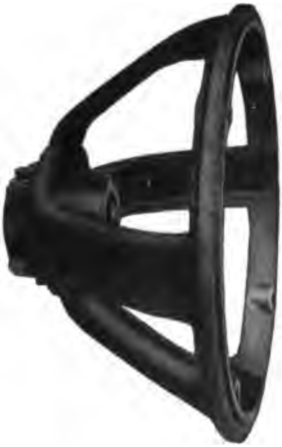
Commutating Field Coil of a Direct Current Motor.



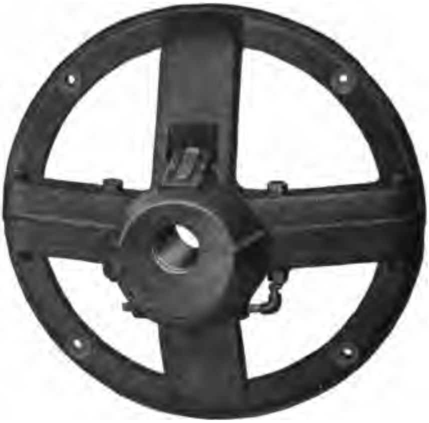
Brush Holders and Brush Yoke for a Four-Pole Motor.



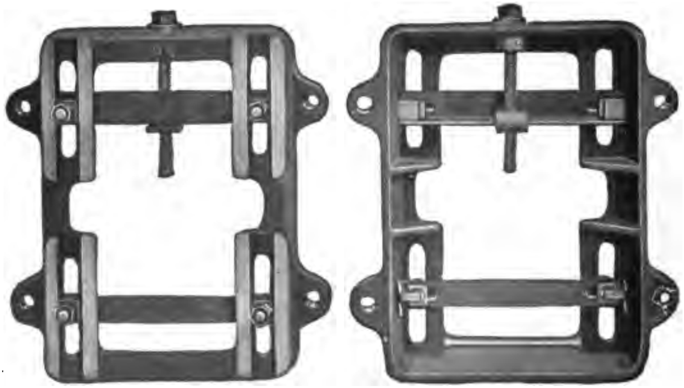
Twin Brush Holder.



Commutator End Shield.

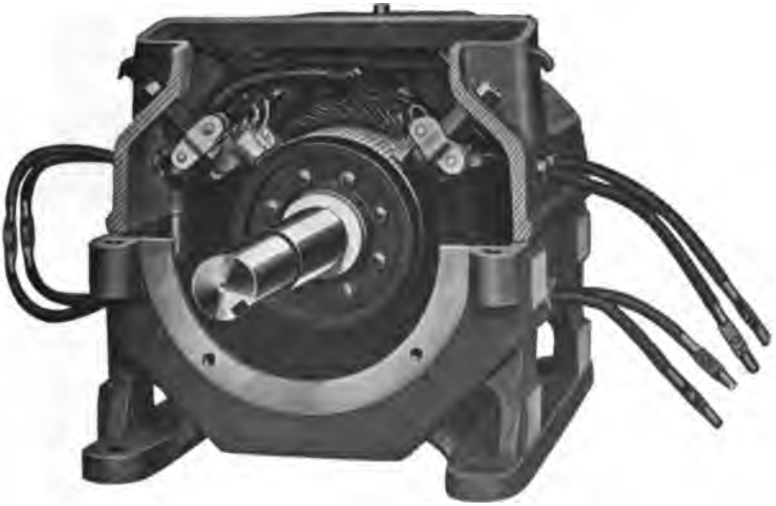


Pulley End Shield.

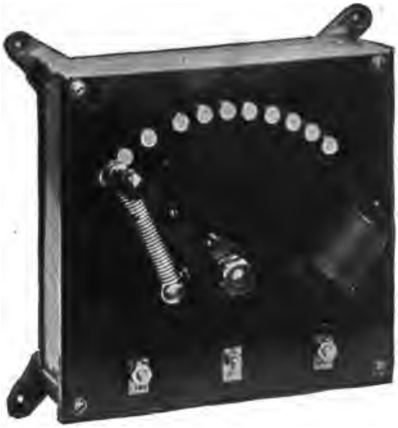


Sliding Base for Wall, Floor or Ceiling Suspension.

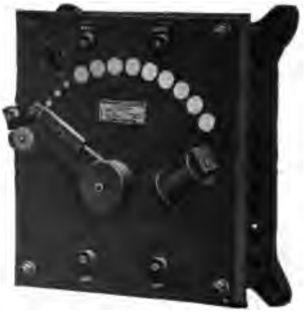
Direct Current Series Motor for Crane Service.



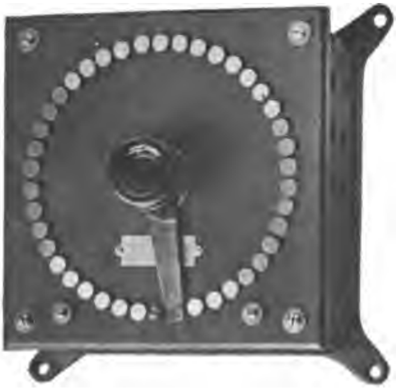
Interior Construction of a Direct Current Series Crane Motor.



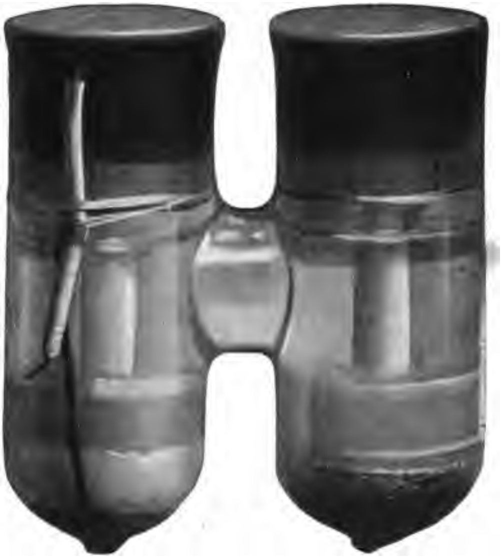
Motor Starting Rheostat for a Shunt Motor.



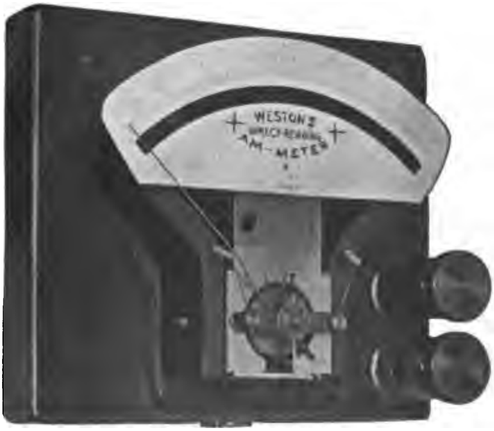
Motor Starting Rheostat for a
Shunt, Series or Compound Motor.



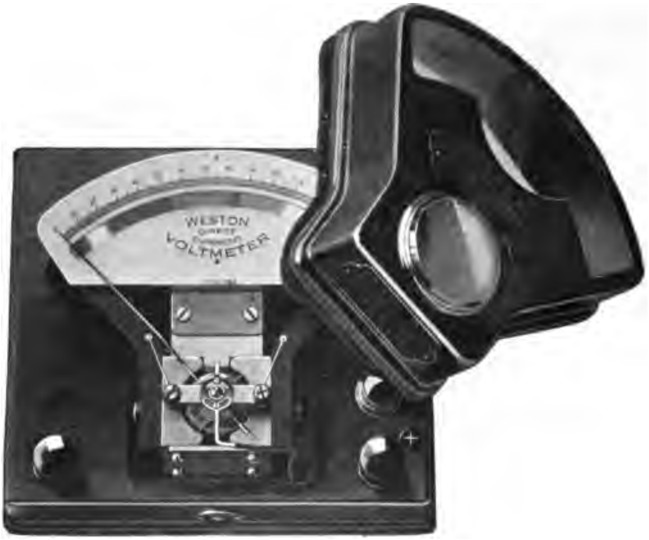
Shunt Field Rheostat.



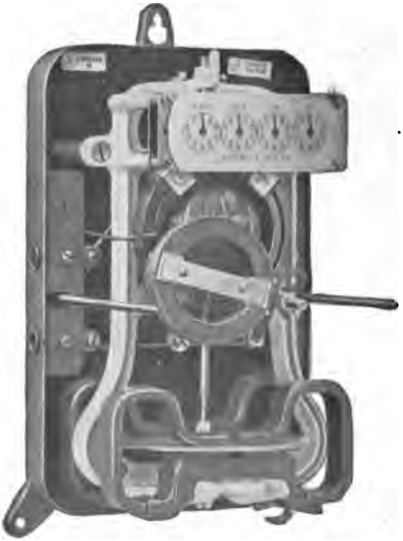
Weston Standard Cell.



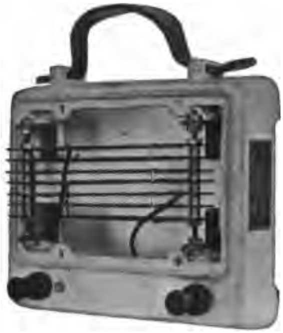
Direct Current Ammeter with Internal Shunt.



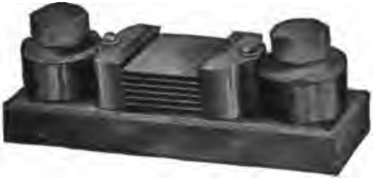
Direct Current Voltmeter.



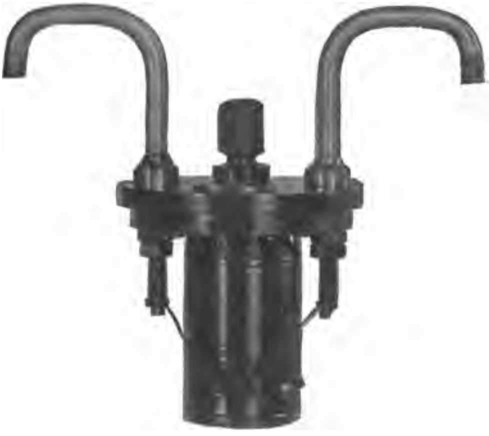
Thomson Direct Current Watthour Meter.



Potential Coil Multiplier.



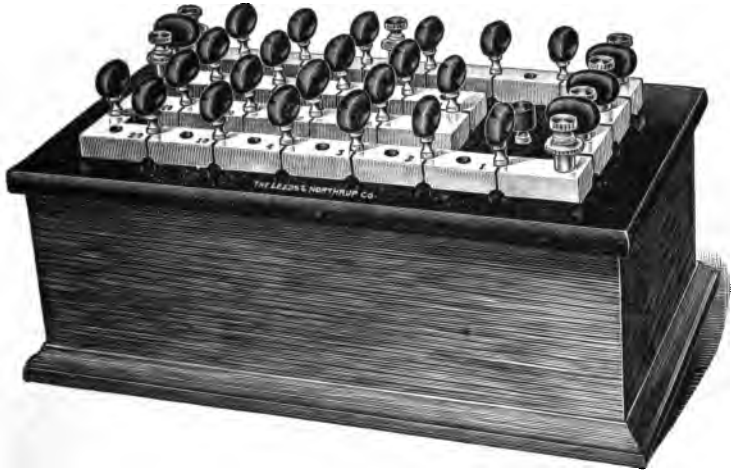
Direct Current Ammeter Shunt.



Standard Resistance — Reichsanstalt Model.

Moving Coil Galvanometer.

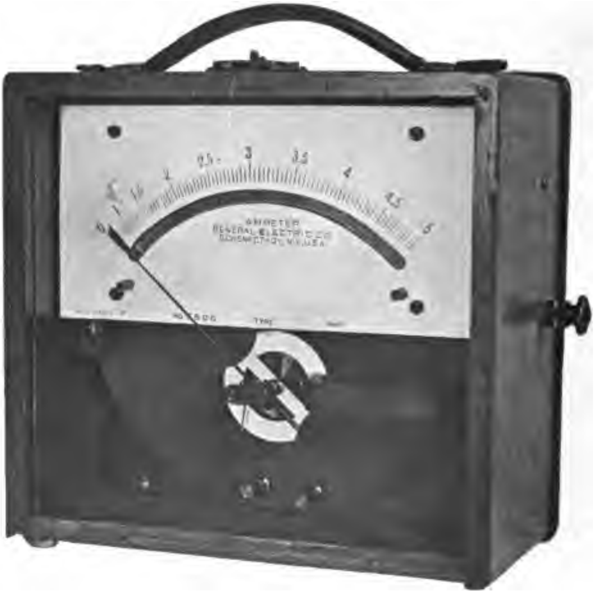
Standard Resistance — N. B. S.
Model.



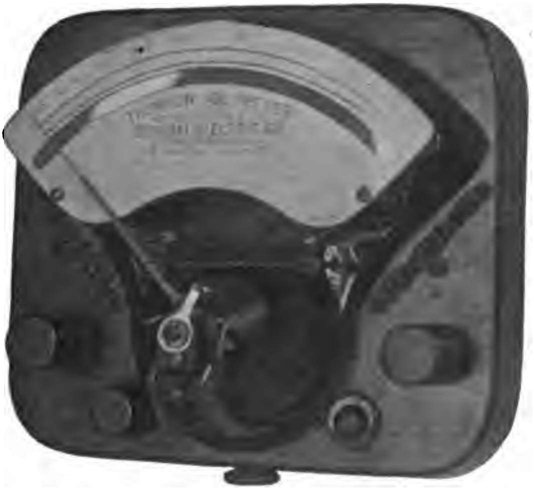
Wheatstone Bridge.

Leeds and Northrup Potentiometer.

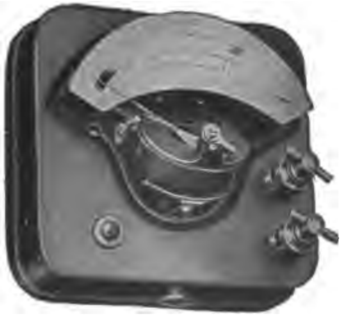
Hot Wire Ammeter.



Interior Construction of a Hot Wire Ammeter.

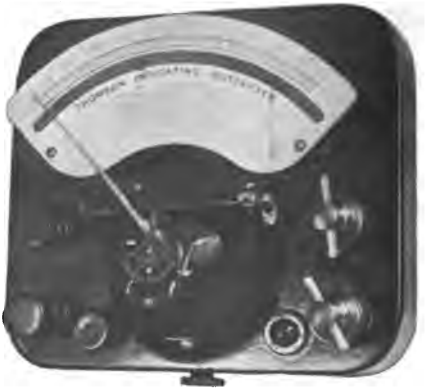


Electrodynamometer Voltmeter.

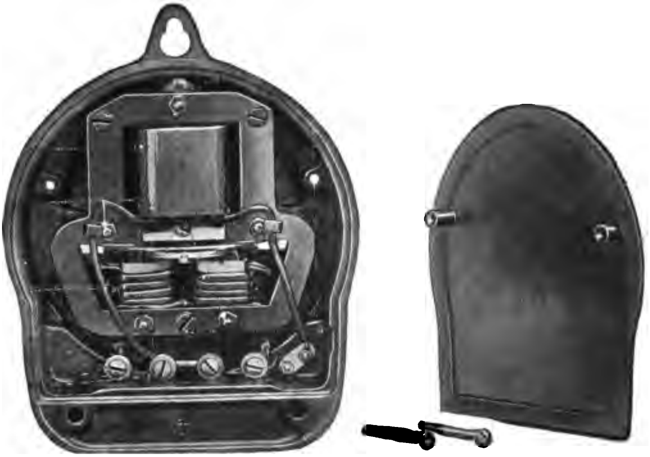


Inclined Coil or Iron Vane Ammeter.

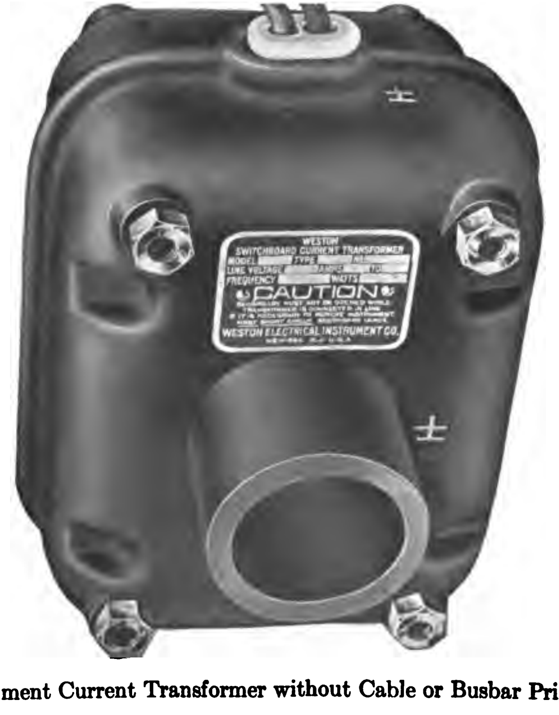
Electrodynamometer Wattmeter.



Interior Construction of an Electro-dynamometer Wattmeter.

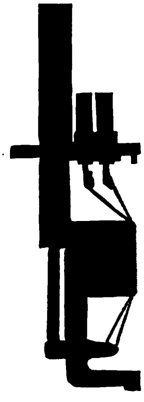


Induction Watthour Meter.

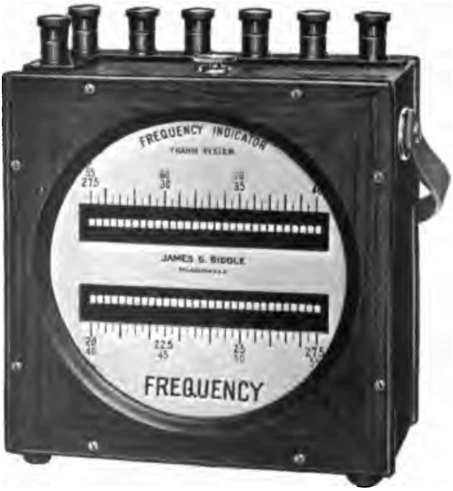

WESTON
SWITCHBOARD CURRENT TRANSFORMER
MODEL
TYPE
NO.
LINE VOLTAGE
AMPS
TO
FREQUENCY
WATTS
CAUTION
WESTON ELECTRICAL INSTRUMENT CO.

Instrument Current Transformer without Cable or Busbar Primary.

Instrument Potential Transformer.

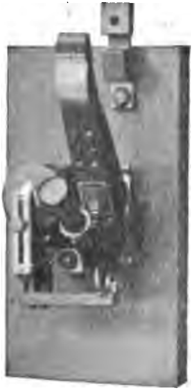


Oscillograph Vibrator.

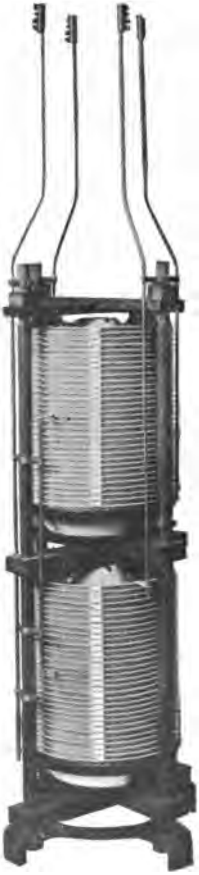


Vibrating Reed Type Frequency Meter.

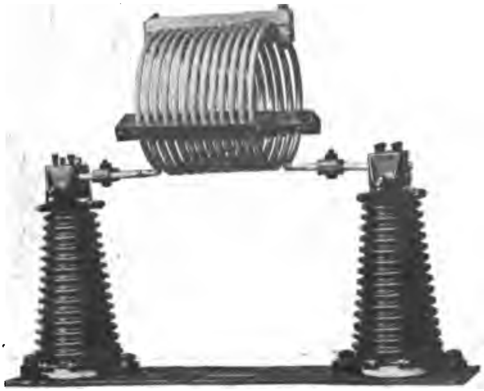
Enclosed Cartridge Fuse.



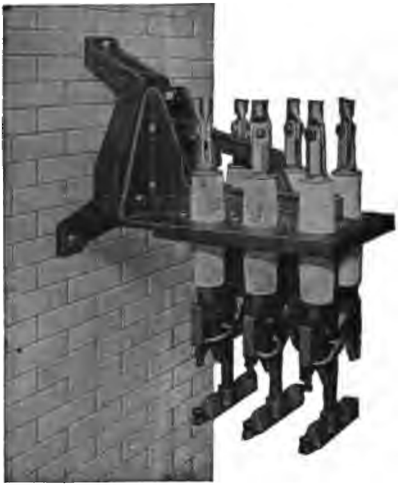
Single-Pole Circuit Breaker with Overload Tripping Coil.



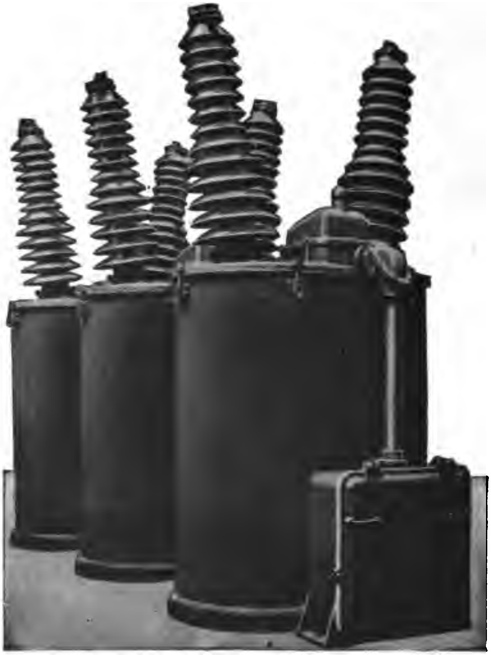
Electrolytic Lightning Arrester.



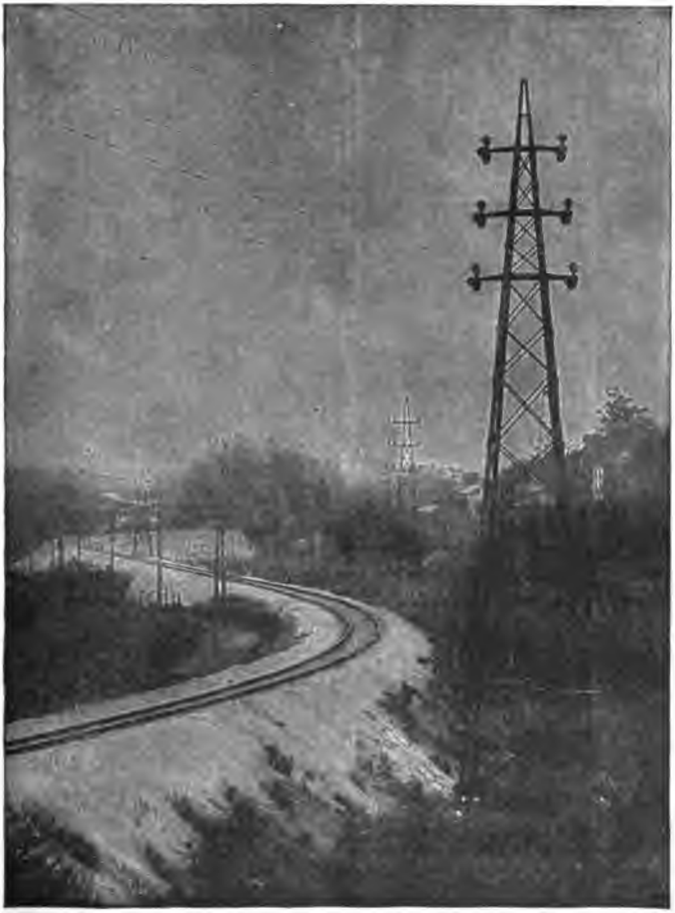
Choke Coil for Generator or Transformer Protection.



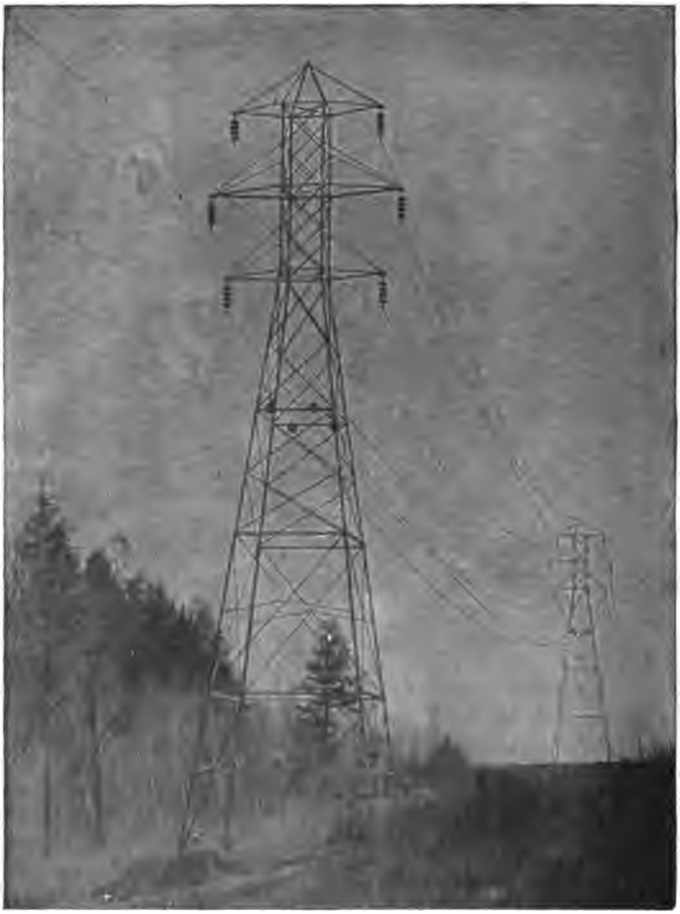
Three-Pole Oil Circuit Breaker with Oil Tank Removed.



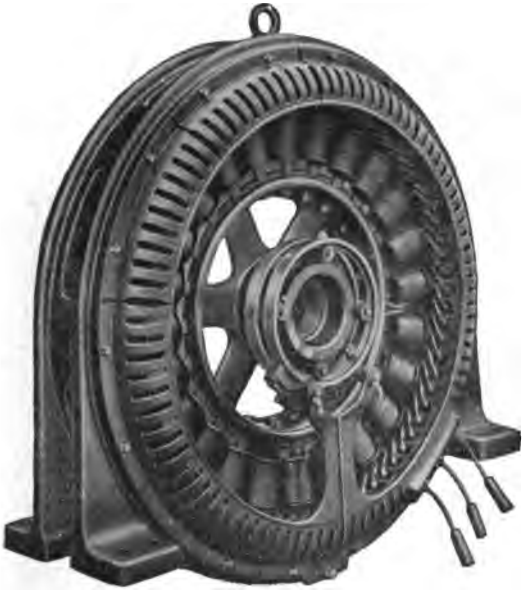
Three-Pole 135,000-Volt Outdoor Oil Circuit Breaker.



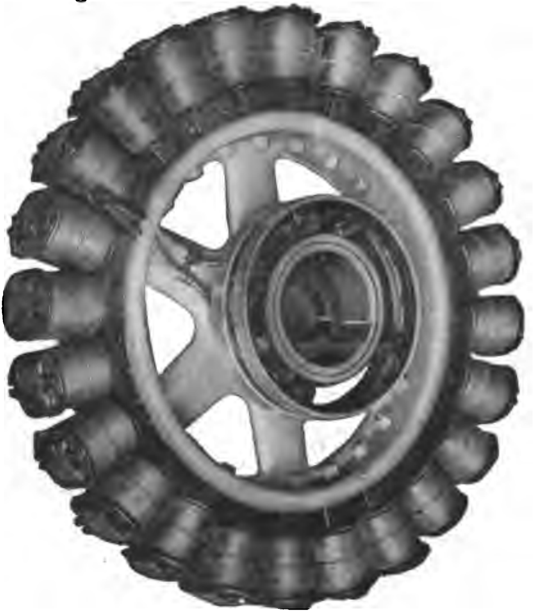
Double Circuit Three-Phase Transmission Line with Pin Type Insulators.



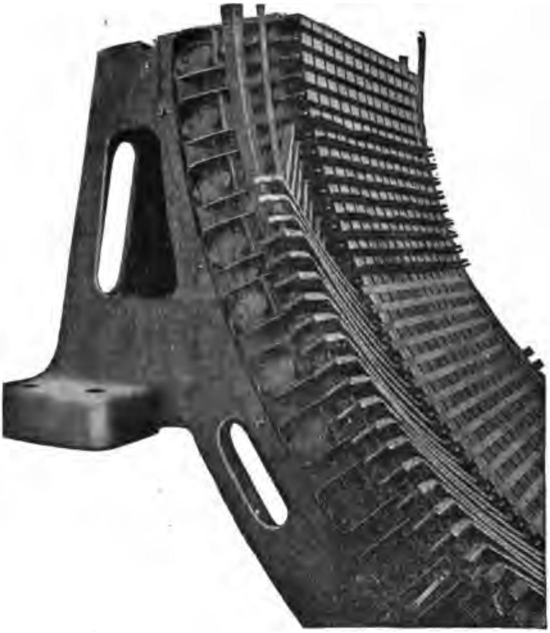
Double Circuit Three-Phase Transmission Line with Suspension Insulators.



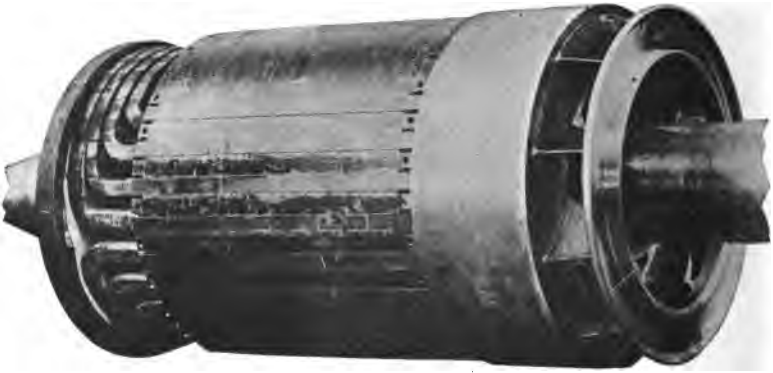
Three-Phase Slow Speed Synchronous Generator with Revolving Field.



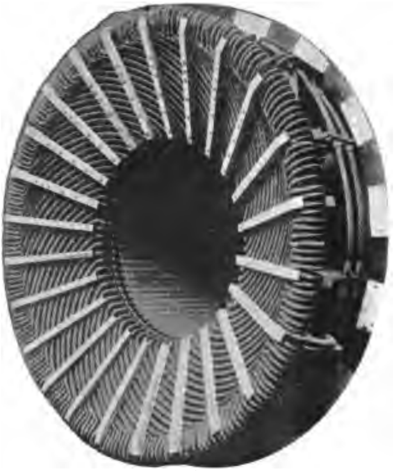
Revolving Field (24 poles) of a Slow Speed Three-Phase Synchronous Generator.



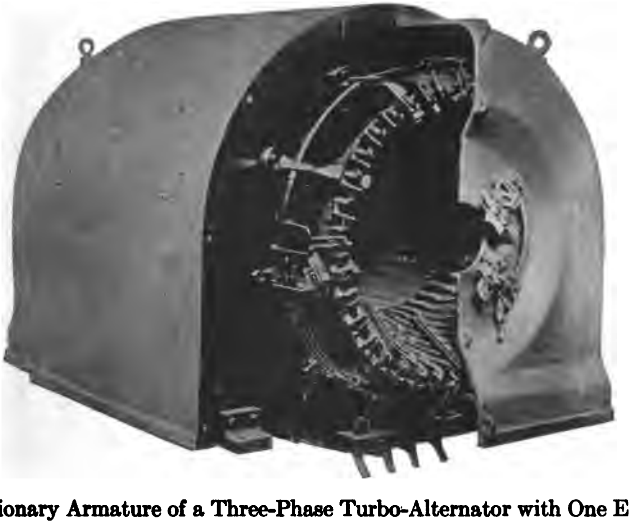
Section of Stationary Armature of a Slow Speed Three-Phase Synchronous Generator showing Method of Assembling Coils.



Two-Pole Revolving Field of a Turbo-Alternator with One End Bell Removed. (Shaft ends not shown.)

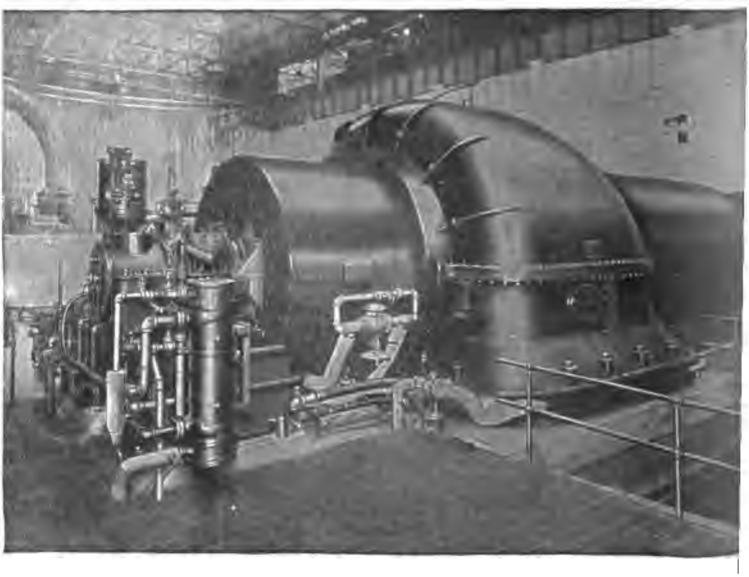


End Connections of the Stationary Armature of a Three-Phase Turbo-Alternator

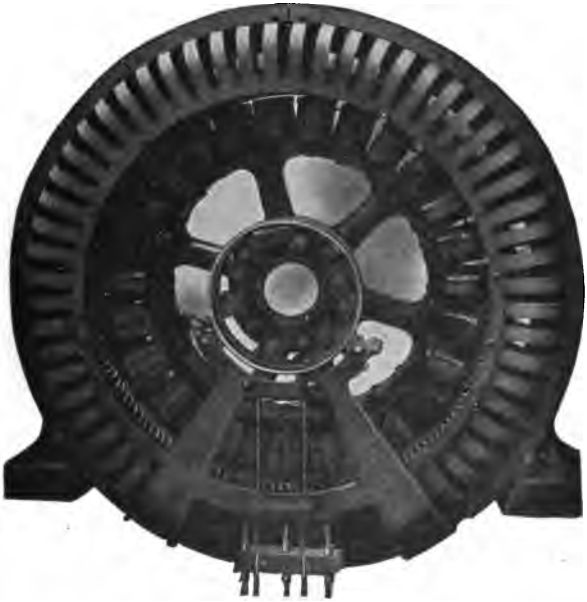


Stationary Armature of a Three-Phase Turbo-Alternator with One End Bell in Place.

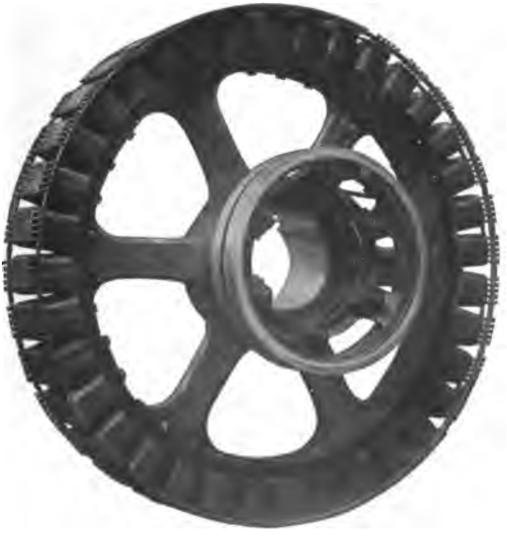
300-Kilowatt Three-Phase Turbo-Alternator.



35,000-Kilowatt Three-Phase Turbo-Alternator.

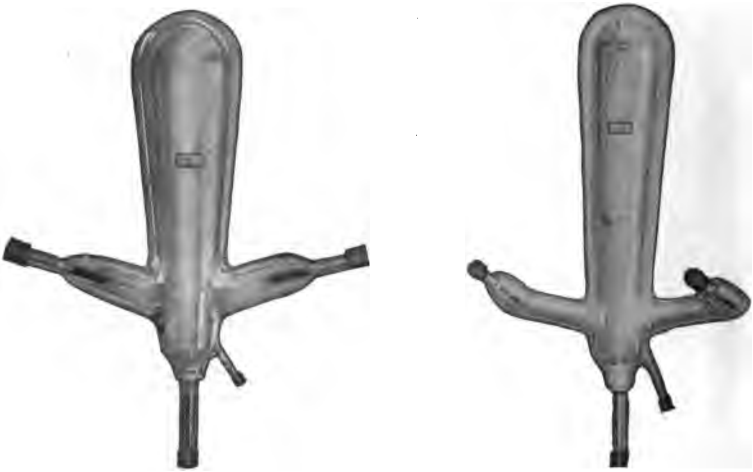


Three-Phase Synchronous Motor.

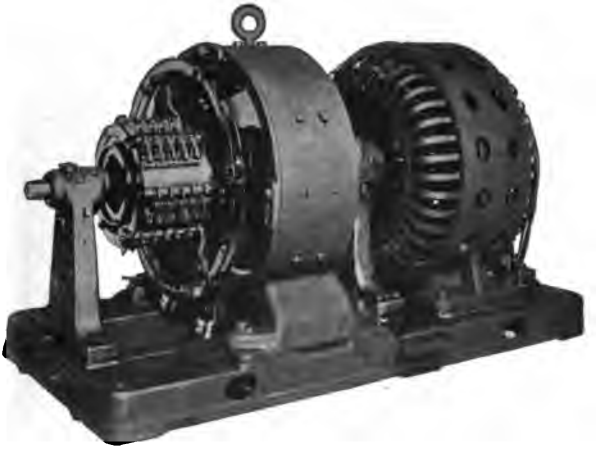


Revolving Field (36 poles) of a Synchronous Motor showing Damper Winding for Starting as an Induction Motor and for Prevention of Hunting.

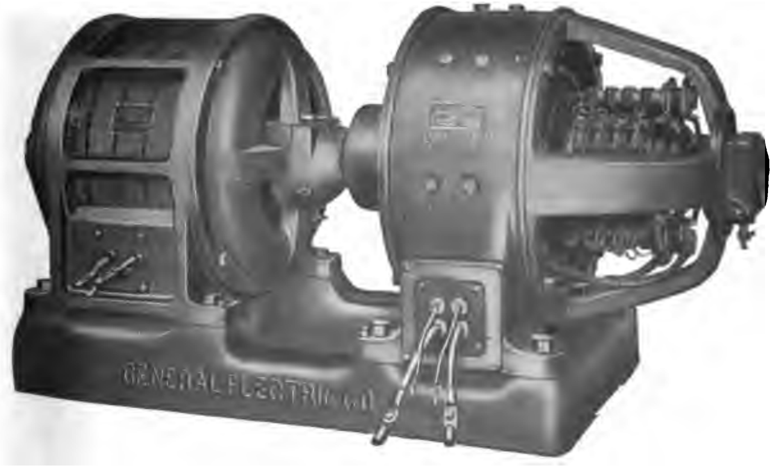
Synchronous Converter.



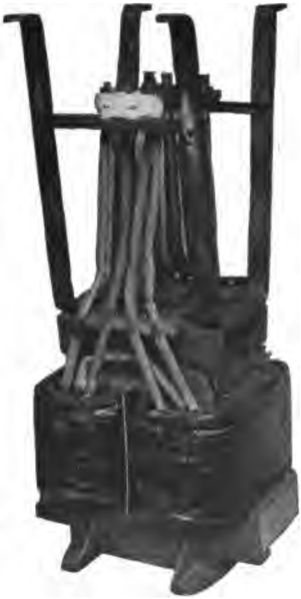
Mercury Arc Rectifier Tubes for 100- and 200-Volt Alternating Current Circuits.



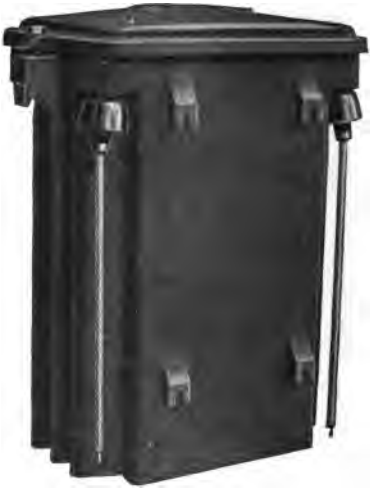
Synchronous Motor-Generator.



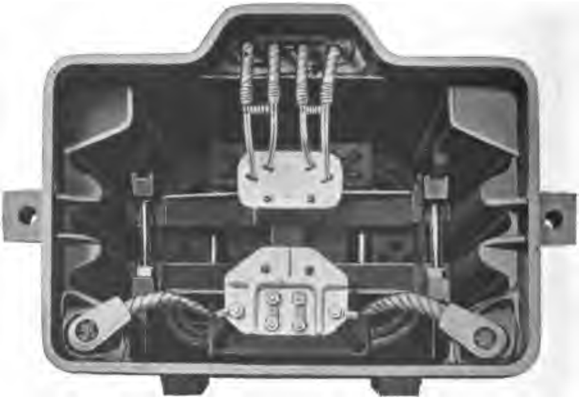
Induction Motor-Generator.



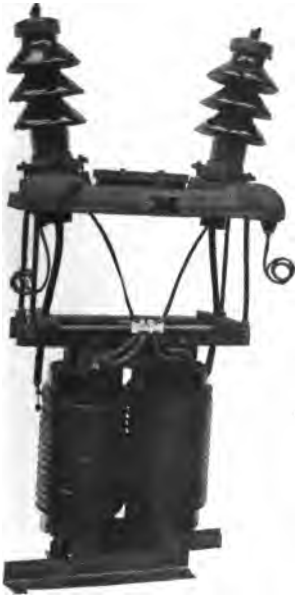
Rectangular Core Type Transformer.



Cast Iron Transformer Tank.

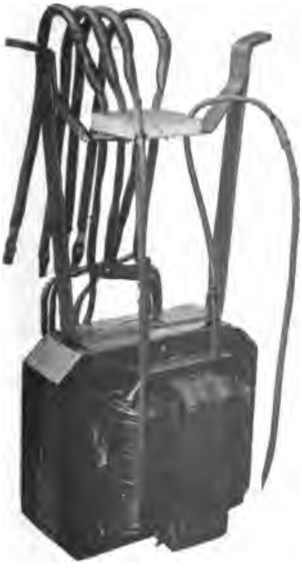


Top View of Rectangular Core Type Transformer in Cast Iron Tank.

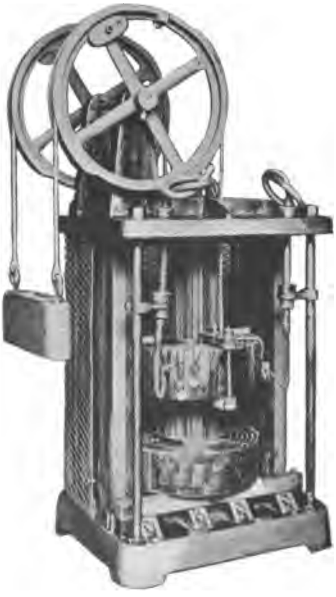


Cruciform Core Type Transformer.

Boiler Iron Transformer Tank.

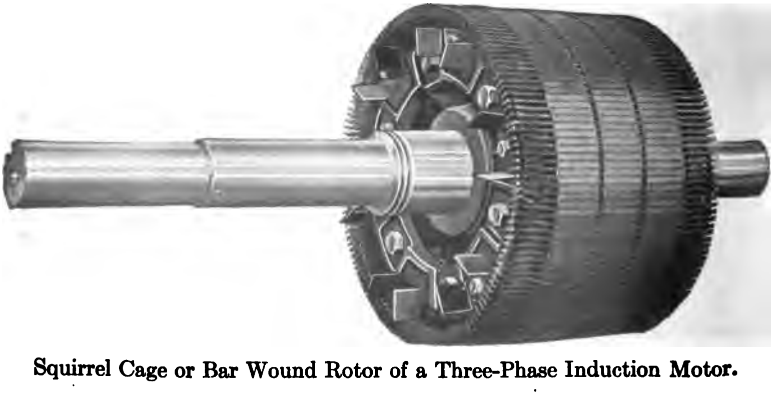


Distributed Shell Type Transformer.

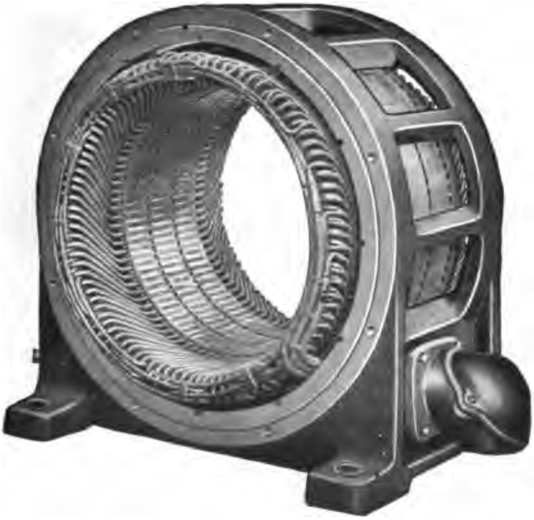


Constant Current Transformer.

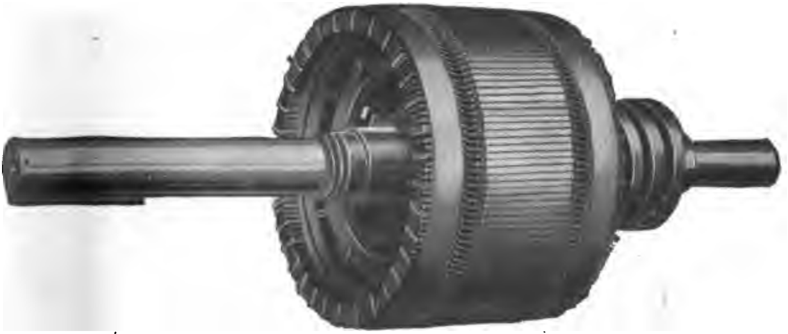
Three-Phase Induction Motor.



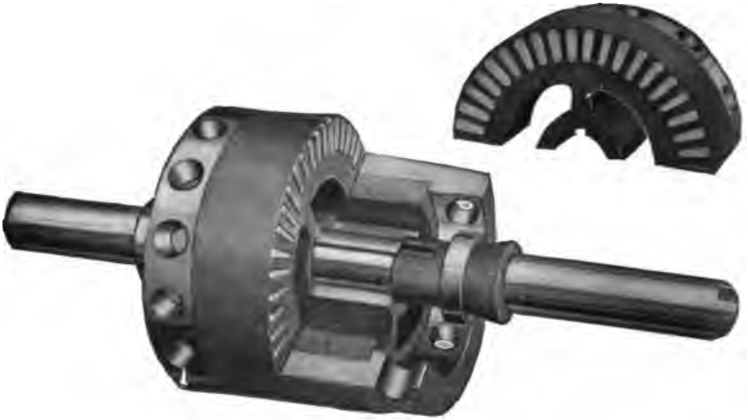
Squirrel Cage or Bar Wound Rotor of a Three-Phase Induction Motor.



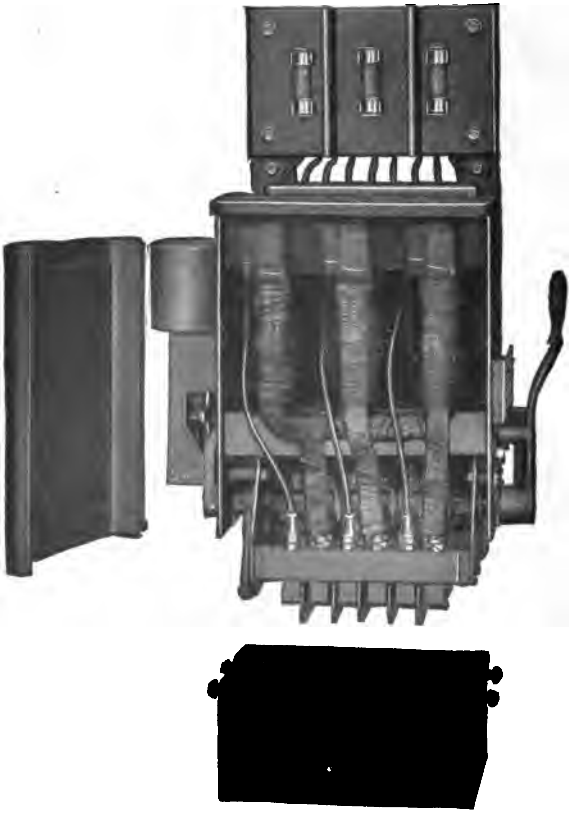
Stator of a Three-Phase Induction Motor.



Wire Wound Rotor of a Three-Phase Induction Motor.



Cast Bar Wound Rotor of a Three-Phase Induction Motor.



Starting Compensator for a Three-Phase Induction Motor with a Bar Wound Rotor.

Stator of a Single-Phase Induction Motor.

Rotor of a Single-Phase Induction Motor.

Series Field and Compensating Winding of a Twelve-Pole Alternating Current Series Commutator Motor for Locomotive Service.

COPPER WIRE TABLE*

Size	Area	Resistance	Weight	Current capacity in amperes	
American Wire Gage	Circular mils	Ohms per 1000 feet at 25° C.	Pounds per 1000 feet	Rubber Insulation	Other Insulations
18	1,620	6.51	4.92	3	5
16	2,580	4.09	7.82	6	10
14	4,110	2.58	12.4	15	20
12	6,530	1.62	19.8	20	25
10	10,400	1.02	31.4	25	30
8	16,500	0.641	50.0	35	50
6	26,300	0.403	79.5	50	70
5	33,100	0.319	100	55	80
4	41,700	0.253	126	70	90
3	52,600	0.205	163	80	100
2	66,400	0.162	205	90	125
1	83,700	0.129	258	100	150
0	106,000	0.102	326	125	200
00	133,000	0.0811	411	150	225
000	168,000	0.0642	518	175	275
0000	212,000	0.0509	653	225	325
....	300,000	0.0360	926	275	400
....	400,000	0.0270	1240	325	500
....	500,000	0.0216	1540	400	600
....	600,000	0.0180	1850	450	680
....	700,000	0.0154	2160	500	760
....	800,000	0.0135	2470	550	840
....	900,000	0.0120	2780	600	920
....	1,000,000	0.0108	3090	650	1000
....	1,100,000	0.00981	3400	690	1080
....	1,200,000	0.00899	3710	730	1150
....	1,300,000	0.00830	4010	770	1220
....	1,400,000	0.00770	4320	810	1290
....	1,500,000	0.00719	4630	850	1360
....	1,600,000	0.00674	4940	890	1430
....	1,700,000	0.00634	5250	930	1490
....	1,800,000	0.00599	5560	970	1550
....	1,900,000	0.00568	5870	1010	1610
....	2,000,000	0.00539	6180	1050	1670

---

* For wires larger than No. 4 the values given are for stranded wires.

RESISTIVITY, TEMPERATURE COEFFICIENT OF RESISTANCE AND DIELECTRIC CONSTANT OF VARIOUS MATERIALS

Resistivity (ρ) and Temperature Coefficient of Resistance (α) of Copper at Various Temperatures

(Resistivity in ohms per mil-foot)

Degs. Cent	ρ	α	Degs. Cent.	ρ	α
0	9.55	0.00427	25	10.6	0.00385
15	10.2	0.00401	30	10.8	0.00378
20	10.4	0.00393	50	11.6	0.00352

Resistivity (ρ_{20}) and Temperature Coefficient of Resistivity (α_{20}) of Certain Conductors at 20° Cent.

(Resistivity in microhms per cm. cube)

Material	ρ_{20}	α_{20}	Material	ρ_{20}	α_{20}
Aluminum........	2.83	0.00390	Iron, steel	12–48	0.003±
Antimony........	38.4–49.8	0.00389	Lead.....	20.0	0.00387
Bismuth..........	117	0.00354	Mercury..	94.1	0.000720
Carbon, brush....	$10^3-6\times10^3$	−0.0009	Nickel....	12.0–13.9	0.00622*
" arc lamp.	$5\times10^3-15\times10^3$	......	Platinum.	9.6	0.00367*
" inc. lamp.	4×10^3	−0.0003	Silver....	1.62	0.00377
Cobalt...........	10.3	0.00325*	Tantalum	15.6	0.00310
Copper...........	1.72	0.00393	Tin......	10.2–12.0	0.00365
Gold.............	2.20	0.00365	Tungsten.	4.9–6.1	0.0047
Iron, cast.........	79–104	......	Zinc......	6.0–6.5	0.00365

* Average value, 0° to 100° Cent.

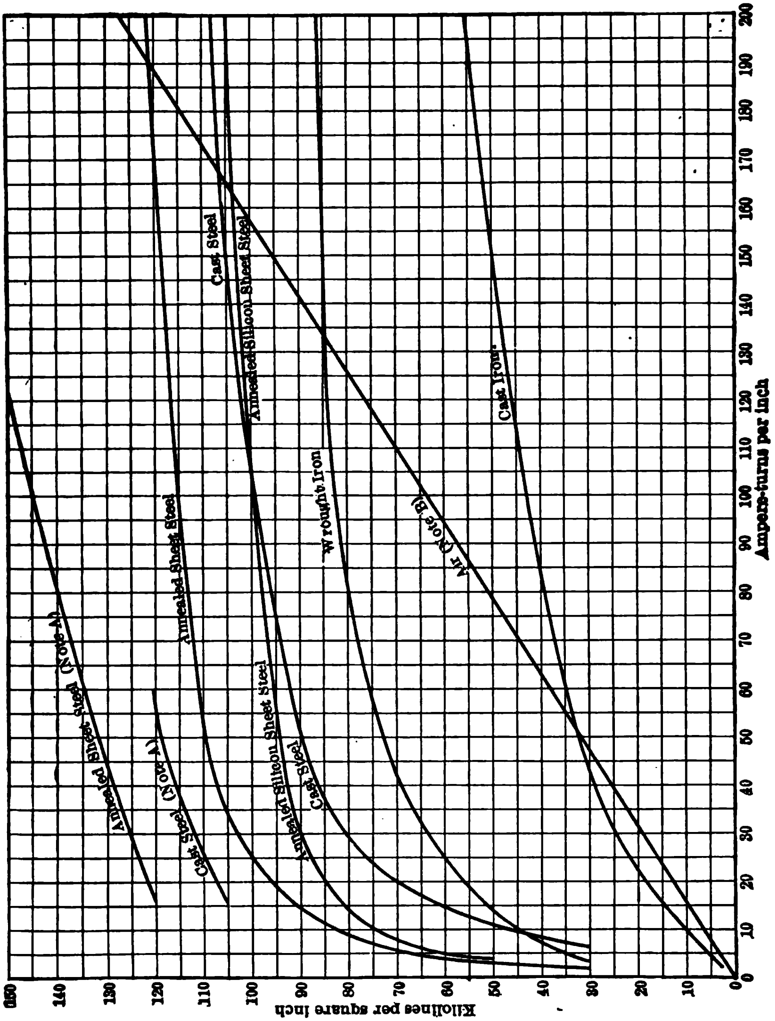
Resistivity (ρ) and Dielectric Constant (k) of Certain Insulators at Room Temperature

(Resistivity in megohms per cm. cube)

Material	ρ	k	Material	ρ	k
Asbestos.....	2×10^5	2.7	Paraffin.....	10^9-10^{11}	1.9–2.3
Celluloid.....	7×10^4	13.3	Porcelain...	10^8-10^9	4.4–6.8
Cellulose.....	10^2-10^4	3.9–7.5	Quartz.....	10^8-10^{11}	4.3–5.1
Glass.........	10^7-10^{12}	5.5–9.1	Rubber.....	$10^8-4\times10^9$	2.3–4.0
Gutta percha.	3×10^4	2.9	Selenium...	0.06	6.1–7.4
Ice...........	720	86	Shellac.....	10^9-10^{10}	2.7–3.8
Marble.......	10^2-10^4	8.3	Silica, fused.	10^8-10^{13}	3.5–3.6
Mica.........	10^6-10^8	2.5–5.9	Slate.......	10^2-10^4	6.6–7.4
Oil, transf.....	$10^4-3\times10^6$	2.0–2.5	Sulphur....	10^{11}	2.2–3.9
Paper........	10^4-10^9	1.7–3.8	Wood.......	10^4-10^7	2.5–7.7

Conversion Factors

Multiply	by	to obtain
Megohms....................	10^6	ohms.
Microhms..................	10^{-6}	ohms.
Microhms per cm. cube.......	6.01	ohms per mil-foot.
Ohms per mil-foot...........	0.166	microhms per cm. cube.



Note A. Multiply abscissa scale by 10.
Note B. Multiply abscissa scale by 200.

Conversion Factors

Multiply	by	to obtain
Ampere-turns per inch........	0.394	ampere-turns per cm.
Ampere-turns per cm.........	2.54	ampere-turns per inch.
Kilolines per sq. inch.........	155	gausses.
Gausses.....................	6.45×10^{-3}	kilolines per sq. inch.

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